



Mathematics
Grade 11 - 12
AP Calculus AB

Dr. Mark Toback, Superintendent
Committee: Amy Giovine
Compliance Update Completed on June 2022

This curriculum may be modified through varying techniques, strategies, and materials as per an individual student's Individualized Educational Plan (IEP)

Approved by the Wayne Township Board of Education at the regular meeting held on November 15, 2018.

**Wayne School District
Curriculum Format**

| | |
|---|--|
| Content Area/ Grade Level/ Course: | Mathematics 11-12 AP Calculus AB |
| Unit Plan Title: | Functions and Limits |
| Time Frame | 20 days |

Anchor Standards/Domain* *i.e: ELA: reading, writing i.e.: Math: Algebra

Number and Quantity

N-Q.A Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Algebra

A-SSE.A Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.

A-SSE.B Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A-CED.A Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Functions

F-IF.B Interpret functions that arise in applications in terms of the context.

F-IF.C Analyze functions using different representations

F-BF.A Build a function that models a relationship between two quantities

F-BF.B Build new functions from existing functions

F-LE.B Interpret expressions for functions in terms of the situation they model

Geometry

G-MG.A Apply geometric concepts in modeling situations

Unit Overview

In calculus, many concepts are developed by first considering a discrete model and then examining the limiting case. This makes the idea of limits essential for developing important ideas in Calculus. Students must have an understanding of limits and be able to compute limits graphically, algebraically, and numerically with the help of a graphing calculator. They should be able to examine one sided limits and limits involving infinity. Students should understand the algebraic procedures for finding limits with indeterminate forms. They should also be able to apply the definition of a limit to

determine continuity of a function. Applications of limits will involve calculating average rates of change and instantaneous rates of change. This will set the foundation for finding the limit definition of the derivative function.

Standard Number(s) * i.e: Math: F.LE.A.4 i.e.: NJLSA.R4.

- N-Q.A.1
- N-Q.A.2
- N-Q.A.3
- A-SSE.A.1
- A-SSE.B.3
- A-CED.A.1
- A-CED.A.2
- A-CED.A.4
- F-IF.B
- F-IF.C
- F-BF.A
- F-LE.
- G-MG.A
- 8.1.12.DA.5
- 8.1.12.DA.6
- 8.1.12.AP.1
- 9.1.12.PB.2
- 9.2.12.CAP.4
- 9.4.12.CI.1
- 9.4.12.CT.2
- 9.4.12.IML.3
- 9.4.12.TL.1
- RST.11-12.3
- RST.9-10.7
- RST.9-10.4
- CRP2
- CRP4
- CRP6
- CRP8
- CRP11

Intended Outcomes - {Essential Questions}

- What is a limit?
- In what real-world situations would the calculation of a limit be useful?
- What is the difference between average and instantaneous speed?

Enduring Understandings

- the relationship between limits and how they describe the behavior of a function

- the difference between average rates of change and instantaneous rates of change

In this unit plan, the following 21st Century themes and skills are addressed.

| Check all that apply. 21 st Century Themes | | Indicate whether these skills are E-Encouraged , T-Taught , or A-Assessed in this unit by marking E , T , A on the line before the appropriate skill. 21 st Century Skills | |
|--|---|--|--|
| <input type="checkbox"/> | Global Awareness | <input type="checkbox"/> | E Creativity and Innovation |
| <input type="checkbox"/> | Environmental Literacy | <input type="checkbox"/> | T Critical Thinking and Problem Solving |
| <input type="checkbox"/> | Health Literacy | <input type="checkbox"/> | A Communication |
| <input type="checkbox"/> | Civic Literacy | <input type="checkbox"/> | E Collaboration |
| <input checked="" type="checkbox"/> | Financial, Economic, Business, and Entrepreneurial Literacy | | |

Student Learning Targets/Objectives (Students will know/Students will understand)

- write an equation and sketch a graph of a line given specific information
- recognize the domain and range of a function and even or odd functions
- interpret and find formulas for piecewise functions and compositions of functions
- identify one-to-one functions
- identify the properties and characteristics of exponential and logarithmic functions and trigonometric functions
- calculate average and instantaneous rates of change (speeds)
- calculate limits graphically for both one- and two-sided limits
- define and calculate limit of a function, if it exists, and apply the properties of limits
- use the Sandwich Theorem to find certain limits indirectly
- find and verify end behavior models for various functions
- calculate limits as $x \rightarrow \pm\infty$ and identify horizontal asymptotes
- use limits of $\pm\infty$ to locate vertical asymptotes
- understand the meaning of a continuous function, remove removable discontinuities by extending or modifying a function
- apply the Intermediate Value Theorem and the properties of algebraic combinations and compositions of continuous functions

Assessments (Pre, Formative, Summative, Other)

*Denote required common assessments with an **

Unit Test
Sample AP Multiple Choice problems
Homework
warm up problems

Teaching and Learning Activities

Activities

Students explore limits at discontinuities in four ways: first, using the table feature on their calculators with decreasing increments; second, using algebraic techniques to

| | |
|-----------------------------------|---|
| | <p>“simplify” the expressions given as formulas; third, using the graph trace feature on their calculators; and fourth, using verbal descriptions of functions written in words to create graphs that match the verbal descriptions.</p> |
| <i>Differentiation Strategies</i> | <p>Incorporate the graphing calculator into lessons to give a visual and numerical interpretation of limits</p> <p>Allow students to work in small groups</p> <p>Provide access to Kahn Academy Videos</p> <p>Provide opportunities for questions</p> |
| <i>Honors</i> | |

Resources

- Finney, Ross L., Franklin D. Demana, Bet K. Waits, and Daniel Kennedy. *Calculus: Graphical, Numerical, Algebraic*, 3rd ed. Boston: Pearson, 2006.
- <http://www.calculus-help.com/>
- [AP Central](#)
- [Kahn Academy](#)

**Wayne School District
Curriculum Format**

| | |
|---|--|
| Content Area/ Grade Level/ Course: | Mathematics 11-12 AP Calculus AB |
| Unit Plan Title: | Derivatives |
| Time Frame | 40 days |

Anchor Standards/Domain* *i.e: ELA: reading, writing i.e.: Math: Algebra

Number and Quantity

N-Q.A Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Algebra

A-SSE.A Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.

A-SSE.B Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A-CED.A Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Functions

- F-IF.B Interpret functions that arise in applications in terms of the context.**

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F-IF.C Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F-BF.A Build a function that models a relationship between two quantities

F-BF.B Build new functions from existing functions

F-LE.B Interpret expressions for functions in terms of the situation they model

Unit Overview

Students will build the definition of the derivative using the concept of limits and use the derivative to compute the instantaneous rate of change of a function. Students will learn the rules of finding derivatives of functions such as the constant multiple rule, product rule, quotient rule, and chain rule. They will also learn how to find the derivative of polynomials, trigonometric functions, logarithmic and exponential functions. They will see the connection between the derivative function and the slope of a curve. The concept of the derivative will be applied to linear motion problems and their application to physics.

Standard Number(s) * i.e: Math: F.LE.A.4 i.e.: NJLSA.R4.

- N-Q.A.1
- N-Q.A.2
- N-Q.A.3
- A-SSE.A.1
- A-SSE.B.3
- A-CED.A.1
- A-CED.A.2
- A-CED.A.4
- F-IF.B.4
- F-IF.C.7
- F-BF.A
- F-LE.B
- CRP2
- CRP4
- CRP6
- CRP8
- CRP11
- 8.1.12.DA.5
- 8.1.12.DA.6
- 8.1.12.AP.1
- 9.1.12.PB.2
- 9.2.12.CAP.4
- 9.4.12.CI.1
- 9.4.12.CT.2
- 9.4.12.IML.3
- 9.4.12.TL.1
- RST.11-12.3

- RST.9-10.7
- RST.9-10.4

Intended Outcomes - {Essential Questions}

- What is the definition of the derivative of a function?
- What is the definition of a derivative of a function at a point?
- How does the derivative function relate to the slope of a curve?
- What are the definitions of left-handed and right-handed derivatives?
- What conditions must be met in order for a derivative to exist?
- What are some common forms of notation of a derivative?
- What does a derivative represent graphically?
- How can you approximate a graph of $f'(x)$ given a table of values for $f(x)$?

Enduring Understandings

- The relationship between a derivative function and the slope of a curve or the instantaneous rate of change of a function
- The relationship between position of an object and its velocity and acceleration as derivative functions
- What it means for the derivative to exist at a point
- How to apply the derivative rules to calculate derivatives of various functions

In this unit plan, the following 21st Century themes and skills are addressed.

| <i>Check all that apply.</i> 21st Century Themes | | <i>Indicate whether these skills are E-Encouraged, T-Taught, or A-Assessed in this unit by marking E, T, A on the line before the appropriate skill.</i> 21st Century Skills | |
|---|---|---|---------------------------------------|
| <input type="checkbox"/> | Global Awareness | <input type="checkbox"/> | Creativity and Innovation |
| <input type="checkbox"/> | Environmental Literacy | <input type="checkbox"/> | Critical Thinking and Problem Solving |
| <input type="checkbox"/> | Health Literacy | <input type="checkbox"/> | Communication |
| <input type="checkbox"/> | Civic Literacy | <input type="checkbox"/> | Collaboration |
| <input checked="" type="checkbox"/> | Financial, Economic, Business, and Entrepreneurial Literacy | | |

Student Learning Targets/Objectives (Students will know/Students will understand)

- calculate slopes and derivatives using the definition of the derivative
- graph a function from the graph of its derivative, graph the derivative of a function from its graph
- find where a function is not differentiable and distinguish between corners, cusps, discontinuities, and vertical tangents
- approximate derivatives numerically and graphically
- use the rules of differentiation to calculate derivatives of polynomial, rational, exponential, logarithmic, and trigonometric functions
- use derivatives to analyze straight line motion and solve other problems involving rates of change
- differentiate composite functions using the Chain Rule
- find the derivative of an implicitly defined curve using implicit differentiation
- calculate higher order derivatives (i.e. second and third derivatives)
- calculate derivatives of functions involving the inverse trigonometric functions
- write the equation of a line tangent to a curve
- write the equation of line normal to a curve
- calculate the derivative at a point on the graphing calculator

Assessments (Pre, Formative, Summative, Other)

*Denote required common assessments with an **

Unit Quizzes/Test

Sample AP Multiple Choice and Open Ended problems

Homework

Teaching and Learning Activities

| | |
|-----------------------------------|---|
| <i>Activities</i> | Students use Desmos to create a slider showing a function and its derivative. The graph shows the tangent line sliding along the curve as its slope values are plotted. Polynomial, rational, exponential, logarithmic, trigonometric, and inverse trigonometric functions are used as the parent function. |
| <i>Differentiation Strategies</i> | Incorporate the graphing calculator into lessons to give a visual and numerical interpretation of limits Allow students to work in small groups Provide access to Kahn Academy Videos Provide opportunities for questions |
| <i>Honors</i> | |

Resources

- Finney, Ross L., Franklin D. Demana, Bet K. Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic, 3rd ed. Boston: Pearson, 2006.
- [desmos.com](https://www.desmos.com)
- [AP Central](https://www.apcentral.collegeboard.org)

| | |
|--|--|
| Content Area/ Grade Level/ Course: | Mathematics 11-12 AP Calculus AB |
| Unit Plan Title: | Applications of Derivatives |
| Time Frame | 30 days |
| Anchor Standards/Domain* *i.e: ELA: reading, writing i.e.: Math: Algebra | |
| Number and Quantity N-Q.A Reason quantitatively and use units to solve problems. 1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. 2. Define appropriate quantities for the purpose of descriptive modeling. 3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | |
| Algebra A-SSE.A Interpret the structure of expressions 1. Interpret expressions that represent a quantity in terms of its context. A-SSE.B Write expressions in equivalent forms to solve problems | |

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A-CED.A Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Functions

F-IF.B Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F-IF.C Analyze functions using different representations

F-BF.A Build a function that models a relationship between two quantities

F-BF.B Build new functions from existing functions

F-LE.B Interpret expressions for functions in terms of the situation they model

Geometry

G-GMD.A Explain volume formulas and use them to solve problems

3. Use Volume formulas for cylinders, pyramids, cones, and spheres to solve problems

G-MG.A Apply geometric concepts in modeling situations

1. Use geometric shapes, their measures, and their properties to describe objects.
2. Apply concepts of density based on area and volume in modeling situations
3. Apply geometric methods to solve design problems

Unit Overview

Students will now be able to apply their knowledge of derivatives and derivative rules to solve real world problems. They will be able to analyze a function (i.e. find intervals of increasing/decreasing, intervals of concavity, extrema) based on information about its first and second derivatives. The idea of locating maximums and minimums will extend to setting up and solving optimization problems. Students will also apply the concept of rates of change to solve related rate problems. Also, they will implement their knowledge of the tangent line to approximate function values through linearization. Applications to physics, business, and geometry will be investigated.

Standard Number(s) * i.e: Math: F.LE.A.4 i.e.: NJLSA.R4.

- N-Q.A.1
- N-Q.A.2
- N-Q.A.3
- A-SSE.A.1
- A-SSE.B.3
- A-CED.A.1
- A-CED.A.2
- A-CED.A.4
- F-IF.B.4
- F-IF.C
- F-BF.A
- F-LE.B

- G-MG.A
- CRP2
- CRP4
- CRP6
- CRP8
- CRP11
- 8.1.12.DA.5
- 8.1.12.DA.6
- 8.1.12.AP.1
- 9.1.12.PB.2
- 9.2.12.CAP.4
- 9.4.12.CI.1
- 9.4.12.CT.2
- 9.4.12.IML.3
- 9.4.12.TL.1
- RST.11-12.3
- RST.9-10.7
- RST.9-10.4

Intended Outcomes - {Essential Questions}

- What information can be found from the graph of the first derivative to help sketch the graph of $f(x)$?
- How are derivative functions used in problem solving when modeling real world situations?
- What does it mean to be “locally linear”?

Enduring Understandings

- A function’s first derivative can be used to analyze the behavior of a function
- The derivative has multiple interpretations and applications to optimization and rates of change
- Tangent lines can be used as approximating functions

In this unit plan, the following 21st Century themes and skills are addressed.

| <p style="text-align: center;"><i>Check all that apply.</i></p> <p style="text-align: center;">21st Century Themes</p> | <p style="text-align: center;"><i>Indicate whether these skills are E-Encouraged, T-Taught, or A-Assessed in this unit by marking E, T, A on the line before the appropriate skill.</i></p> <p style="text-align: center;">21st Century Skills</p> |
|--|--|
| <input type="checkbox"/> | <input type="checkbox"/> Global Awareness |
| <input type="checkbox"/> | <input type="checkbox"/> Environmental Literacy |
| <input type="checkbox"/> | <input type="checkbox"/> Health Literacy |
| <input type="checkbox"/> | <input type="checkbox"/> E Creativity and Innovation |
| <input type="checkbox"/> | <input type="checkbox"/> T Critical Thinking and Problem Solving |
| <input type="checkbox"/> | <input type="checkbox"/> A Communication |

| |
|---|
| |
| X |

Civic Literacy
Financial, Economic, Business, and Entrepreneurial Literacy

| |
|---|
| E |
|---|

Collaboration

Student Learning Targets/Objectives (Students will know/Students will understand)

- locate critical values of a function using derivatives
- apply the Mean Value Theorem and Extreme Value Theorem
- use the first and second derivative tests to determine local and absolute extreme values of a function
- determine concavity of a function and locate points of inflection by analyzing second derivatives
- graph a function using first and second derivative information
- solve application problems involving finding minimum or maximum values of a function
- find linearizations and apply the result when approximating functional values
- calculate the error in approximation when using a tangent-line approximation
- solve related rate problems
- apply L'Hopital's rule to evaluate limits with indeterminate forms

Assessments (Pre, Formative, Summative, Other) *Denote required common assessments with an **

Unit Quizzes/Test
 Sample AP Multiple Choice and Open Ended problems
 Homework
 Warm Up Problems

Teaching and Learning Activities

| | |
|-----------------------------------|--|
| <i>Activities</i> | Using the graph of the derivative $f'(x)$, students determine key features of $f(x)$ (such as increasing/decreasing intervals, local extrema, points of inflection, and concavity intervals) and create a graph of $f(x)$. |
| <i>Differentiation Strategies</i> | Incorporate the graphing calculator into lessons to give a visual and numerical interpretation of concepts Allow students to work in small groups Provide access to Kahn Academy Videos Provide opportunities for questions |
| <i>Honors</i> | |

Resources

- Finney, Ross L., Franklin D. Demana, Bet K. Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic, 3rd ed. Boston: Pearson, 2006.
- [Texas Instruments for AP](#)
- [AP Central](#)
- [Kahn Academy](#)

| | |
|---|--|
| Content Area/ Grade Level/ Course: | Mathematics 11-12 AP Calculus AB |
| Unit Plan Title: | The Definite Integral |
| Time Frame | 30 days |

Anchor Standards/Domain* *i.e: ELA: reading, writing i.e.: Math: Algebra

Number and Quantity

N-Q.A Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Algebra

A-SSE.A Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.

A-SSE.B Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A-CED.A Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Functions

F-IF.B Interpret functions that arise in applications in terms of the context.

F-IF.C Analyze functions using different representations

F-BF.A Build a function that models a relationship between two quantities

F-BF.B Build new functions from existing functions

F-LE.B Interpret expressions for functions in terms of the situation they model

Geometry

G-MG.A Apply geometric concepts in modeling situations

Unit Overview

Students will learn the definition of the definite integral as a Riemann sum and be able to approximate the definite integral using various methods. They will be able to use approximating methods such as the Rectangular Approximating Method and the Trapezoid Rule. They will also learn to use geometry formulas to calculate the signed area between the graph of $f(x)$ and the x -axis. Students will also familiarize themselves with basic techniques of integration and the properties of the definite integral. The definite integral will be thought of as an accumulating function and students will be able to evaluate integrals by the Fundamental Theorem of Calculus. Students will also apply the Fundamental Theorem of Calculus to see the connection between integration and differentiation of functions.

Standard Number(s) * i.e: Math: F.LE.A.4 i.e.: NJLSA.R4.

- N-Q.A.1
- N-Q.A.2
- N-Q.A.3
- A-SSE.A.1
- A-SSE.B.3
- A-CED.A.1
- A-CED.A.2
- A-CED.A.4
- F-IF.B
- F-IF.C
- F-BF.A
- F-LE.B
- G-MG.A
- CRP2
- CRP4
- CRP6
- CRP8
- CRP11
- 8.1.12.DA.5
- 8.1.12.DA.6
- 8.1.12.AP.1
- 9.1.12.PB.2
- 9.2.12.CAP.4
- 9.4.12.CI.1
- 9.4.12.CT.2
- 9.4.12.IML.3
- 9.4.12.TL.1
- RST.11-12.3
- RST.9-10.7
- RST.9-10.4

Intended Outcomes - {Essential Questions}

- What are the different ways you can use Riemann Sums to approximate area under a curve and how is that related to integration?
- What is a definite integral?
- How are definite integrals like limits?
- How do you use the fundamental theorem of calculus to evaluate definite integrals and anti- derivatives?
- How is the integrand of a definite integral related to the integral function?

Enduring Understandings

- Antidifferentiation is the inverse process of differentiation
- The definite integral of a function over an interval is the limit of a Riemann sum

- The definite integral of a function has many interpretations and applications involving accumulation
- The Fundamental Theorem of Calculus can be applied to evaluate definite integrals

In this unit plan, the following 21st Century themes and skills are addressed.

| Check all that apply. 21 st Century Themes | | Indicate whether these skills are E-Encouraged , T-Taught , or A-Assessed in this unit by marking E, T, A on the line before the appropriate skill. 21 st Century Skills | |
|--|---|--|---------------------------------------|
| <input type="checkbox"/> | Global Awareness | <input type="checkbox"/> | Creativity and Innovation |
| <input type="checkbox"/> | Environmental Literacy | <input type="checkbox"/> | Critical Thinking and Problem Solving |
| <input type="checkbox"/> | Health Literacy | <input type="checkbox"/> | Communication |
| <input type="checkbox"/> | Civic Literacy | <input type="checkbox"/> | Collaboration |
| <input checked="" type="checkbox"/> | Financial, Economic, Business, and Entrepreneurial Literacy | | |

Student Learning Targets/Objectives (Students will know/Students will understand)

- approximate the area under the graph of a non-negative continuous function using rectangular approximation methods (RAM)
- interpret the area under the graph as a net accumulation of a rate of change
- express the area under a curve as a definite integral and as a limit of Riemann sums and calculate the area
- apply the rules for definite integrals and find the average value of a function over a closed interval
- apply and understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus
- define a function using a definite integral and calculate its derivatives
- evaluate definite integrals using the Fundamental Theorem of Calculus
- approximate the definite integral using the Trapezoidal Rule
- construct antiderivatives using the Fundamental Theorem of Calculus

Assessments (Pre, Formative, Summative, Other)

*Denote required common assessments with an **

Unit Quizzes/Test
Sample AP Multiple Choice and Open Ended problems
Homework

Teaching and Learning Activities

Activities

Students write an expression for an approximation of the area between the x axis and the graph of $f(x)$ for a particular function given as a formula on a specified interval as a left, right, and midpoint Riemann sum using n subdivisions. They then use a Desmos graph with slider to explore sums. The file superimposes rectangular areas on the graph of $f(x)$, showing the sum value. Also, students will write limits of their Riemann sums as n goes to infinity, then identify each as a definite integral, and use the Fundamental Theorem of Calculus to evaluate the integral.

Students will solve an AP Open Ended

| | |
|-----------------------------------|--|
| <i>Differentiation Strategies</i> | Incorporate the graphing calculator into lessons to give a visual and numerical interpretation of concepts Allow students to work in small groups Provide access to Kahn Academy Videos Provide opportunities for questions |
| <i>Honors</i> | |

Resources

- Finney, Ross L., Franklin D. Demana, Bet K. Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic, 3rd ed. Boston: Pearson, 2006.
- [Texas Instruments for AP](#)
- [AP Central](#)
- [Desmos](#)
- [Kahn Academy](#)

**Wayne School District
Curriculum Format**

| | |
|---|--|
| Content Area/ Grade Level/ Course: | Mathematics 11-12 AP Calculus AB |
| Unit Plan Title: | Applications of the Definite Integral and Differential Equations |
| Time Frame | 35 Days |

Anchor Standards/Domain* *i.e: ELA: reading, writing i.e.: Math: Algebra

Number and Quantity

N-Q.A Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Algebra

A-SSE.A Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.

A-SSE.B Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A-CED.A Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Functions

F-IF.B Interpret functions that arise in applications in terms of the context.

F-IF.C Analyze functions using different representations

F-BF.A Build a function that models a relationship between two quantities

F-BF.B Build new functions from existing functions

F-LE.B Interpret expressions for functions in terms of the situation they model

Geometry

G-MG.A Apply geometric concepts in modeling situations

Unit Overview

Once students are familiar with the definite integral and antidifferentiation techniques, they will explore applications to finding areas in the plane and volumes of solids. They will revisit one dimensional motion and how definite integrals can calculate displacement and distance traveled. Students will also discover how to solve differential equations graphically through slope fields and algebraically through initial value problems. Students will learn more advanced tools of integration such as u-substitution and integration by parts.

Standard Number(s) * i.e: **Math: F.LE.A.4** i.e.: **NJSLSA.R4.**

- **N-Q.A.1**
- **N-Q.A.2**
- **N-Q.A.3**
- **A-SSE.A.1**
- **A-SSE.B.3**
- **A-CED.A.1**
- **A-CED.A.2**
- **A-CED.A.4**
- **F-IF.B**
- **F-IF.C**
- **F-BF.A**
- **F-LE.B**
- **G-MG.A**
- **CRP2**
- **CRP4**
- **CRP6**
- **CRP8**
- **CRP11**
- **8.1.12.DA.5**
- **8.1.12.DA.6**
- **8.1.12.AP.1**
- **9.1.12.PB.2**
- **9.2.12.CAP.4**
- **9.4.12.CI.1**
- **9.4.12.CT.2**
- **9.4.12.IML.3**
- **9.4.12.TL.1**
- **RST.11-12.3**
- **RST.9-10.7**
- **RST.9-10.4**

Intended Outcomes - {Essential Questions}

- How does a slope field help approximate a function?
- How do you solve a differential equation?
- When can u-substitution be applied and how can we use it to integrate?
- How are differential equations used to solve real-world problems?
- How can a definite integral calculate the area and volumes of uncommon shapes and solids?

Enduring Understandings

- The solution to a differential equation can be represented graphically through a slope field or algebraically through antiderivatives
- The definite integral can be applied to motion problems to calculate displacement and distance traveled
- Separation of variables is required when solving a differential equation in terms of x and y
- Volume can be calculated by integrating the area of cross-sections over an interval

In this unit plan, the following 21st Century themes and skills are addressed.

| <i>Check all that apply.</i> 21st Century Themes | | <i>Indicate whether these skills are E-Encouraged, T-Taught, or A-Assessed in this unit by marking E, T, A on the line before the appropriate skill.</i> 21st Century Skills | |
|---|---|---|---------------------------------------|
| <input type="checkbox"/> | Global Awareness | <input type="checkbox"/> E | Creativity and Innovation |
| <input type="checkbox"/> | Environmental Literacy | <input type="checkbox"/> T | Critical Thinking and Problem Solving |
| <input type="checkbox"/> | Health Literacy | <input type="checkbox"/> A | Communication |
| <input type="checkbox"/> | Civic Literacy | <input type="checkbox"/> E | Collaboration |
| <input checked="" type="checkbox"/> | Financial, Economic, Business, and Entrepreneurial Literacy | | |

Student Learning Targets/Objectives (Students will know/Students will understand)

- solve initial value problems given a derivative and initial condition
- construct slope fields using technology and interpret slope fields as visualizations of differential equations
- solve differential equations of the form $dy/dx = f(x)$
- compute indefinite and definite integrals by the method of substitution
- compute indefinite and definite integrals by the method of integration by parts
- solve differential equations of the form $dy/dx = f(x) \cdot g(y)$ in which the variables are separable
- solve problems involving exponential growth in a variety of applications
- to solve problems in which a rate is integrated to find the net change over time in a variety of applications
- use a definite integral to calculate total distance traveled by a particle over an interval
- use integration to calculate areas of regions in a plane

- use integration by slicing to calculate volumes of solids with known cross sectional areas or formed by rotation of a bounded region about a given axis

Assessments (Pre, Formative, Summative, Other)

*Denote required common assessments with an **

Unit Quizzes/Test

Sample AP Multiple Choice and Open Ended problems

Homework

Teaching and Learning Activities

| | |
|-----------------------------------|--|
| <i>Activities</i> | To visualize the steps necessary to find the volume of a solid with known cross sections, students build a physical model with foam board of weighted paper to construct several cross sections. |
| <i>Differentiation Strategies</i> | Incorporate the graphing calculator into lessons to give a visual and numerical interpretation of concepts Allow students to work in small groups Provide access to Kahn Academy Videos Provide opportunities for questions |
| <i>Honors</i> | |

Resources

- [Video Demonstration for Cross Sectional Volumes](#)
- Finney, Ross L., Franklin D. Demana, Bet K. Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic, 3rd ed. Boston: Pearson, 2006.
- [Texas Instruments for AP](#)
- [AP Central](#)
- [Desmos](#)
- [Kahn Academy](#)