Mathematics
Grade 12
AP Calculus BC

Dr. Mark Toback, Superintendent
Committee: Amy Giovine
Compliance Update Completed on June 2022

This curriculum may be modified through varying techniques, strategies, and materials as per an individual student's Individualized Educational Plan (IEP)

Approved by the Wayne Township Board of Education at the regular meeting held on November 15, 2018.

Content Area/ Grade Level/ Course:	Mathematics 12 AP Calculus BC
Unit Plan Title:	Functions and Limits
Time Frame	10 days

Anchor Standards/Domain*   *i.e: ELA: reading, writing i.e.: Math: Algebra

Number and Quantity

   N-Q.A Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

2. Define appropriate quantities for the purpose of descriptive modeling.

3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Algebra

   A-SSE.A Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.

   A-SSE.B Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

   A-CED.A Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Functions

   F-IF.B Interpret functions that arise in applications in terms of the context.

   F-IF.C Analyze functions using different representations

   F-BF.A Build a function that models a relationship between two quantities

   F-BF.B   Build new functions from existing functions

   F-LE.B Interpret expressions for functions in terms of the situation they model

Geometry

   G-MG.A Apply geometric concepts in modeling situations

Unit Overview

In calculus, many concepts are developed by first considering a discrete model and then examining the limiting case. This makes the idea of limits essential for developing important ideas in Calculus. Students must have an understanding of limits and be able to compute limits graphically, algebraically, and numerically with the help of a graphing calculator. They should be able to examine one sided limits and limits involving infinity. Students should understand the algebraic

procedures for finding limits with indeterminate forms. They should also be able to apply the definition of a limit to determine continuity of a function. Applications of limits will involve calculating average rates of change and instantaneous rates of change. This will set the foundation for finding the limit definition of the derivative function.

Standard Number(s)   * i.e: Math: F.LE.A.4   i.e.: NJSLSA.R4.

- **N-Q.A.1**
- **N-Q.A.2**
- **N-Q.A.3**
- **A-SSE.A.1**
- **A-SSE.B.3**
- **A-CED.A.1**
- **A-CED.A.2**
- **A-CED.A.4**
- **F-IF.B**
- **F-IF.C**
- **F-BF.A**
- **F-LE.B**
- **G-MG.A**
- **CRP2**
- **CRP4**
- **CRP6**
- **CRP8**
- **CRP11**
- **8.1.12.DA.5**
- **8.1.12.DA.6**
- **8.1.12.AP.1**
- **9.1.12.PB.2**
- **9.2.12.CAP.4**
- **9.4.12.CI.1**
- **9.4.12.CT.2**
- **9.4.12.IML.3**
- **9.4.12.TL.1**
- **RST.11-12.3**
- **RST.9-10.7**
- **RST.9-10.4**

Intended Outcomes - {Essential Questions}

- What is a limit?
- In what real-world situations would the calculation of a limit be useful?
- What is the difference between average and instantaneous speed?

Enduring Understandings

- the relationship between limits and how they describe the behavior of a function
- the difference between average rates of change and instantaneous rates of change

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In this unit plan, the following 21st Century themes and skills are addressed.

Check all that apply. **21st Century Themes**			*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* **21st Century Skills**	
		Global Awareness	E	**Creativity and Innovation**
		Environmental Literacy	T	**Critical Thinking and Problem Solving**
		Health Literacy	A	**Communication**
		Civic Literacy	E	**Collaboration**
X		**Financial, Economic, Business, and Entrepreneurial Literacy**		

Student Learning Targets/Objectives (Students will know/Students will understand)

- write an equation and sketch a graph of a line given specific information
- recognize the domain and range of a function and even or odd functions
- interpret and find formulas for piecewise functions and compositions of functions
- identify one-to-one functions
- identify the properties and characteristics of exponential and logarithmic functions and trigonometric functions
- calculate average and instantaneous rates of change (speeds)
- calculate limits graphically for both one- and two-sided limits
- define and calculate limit of a function, if it exists, and apply the properties of limits
- use the Sandwich Theorem to find certain limits indirectly
- find and verify end behavior models for various functions
- calculate limits as $x \to \pm\infty$ and identify horizontal asymptotes
- use limits of $\pm\infty$ to locate vertical asymptotes
- understand the meaning of a continuous function, remove removable discontinuities by extending or modifying a function
- apply the Intermediate Value Theorem and the properties of algebraic combinations and compositions of continuous functions

Assessments (Pre, Formative, Summative, Other)   *Denote required common assessments with an *

Unit Test/Quiz
Sample AP Multiple Choice problems

Homework	
Warm up problems	

Teaching and Learning Activities	
Activities	Students explore limits at discontinuities in four ways: first, using the table feature on their calculators with decreasing increments; second, using algebraic techniques to "simplify" the expressions given as formulas; third, using the graph trace feature on their calculators; and fourth, using verbal descriptions of functions written in words to create graphs that match the verbal descriptions.
Differentiation Strategies	Incorporate the graphing calculator into lessons to give a visual and numerical interpretation of limits Allow students to work in small groups Provide access to Kahn Academy Videos Provide opportunities for questions
Honors	

Resources
● Finney, Ross L., Franklin D. Demana, Bet K. Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic, 3rd ed. Boston: Pearson, 2006. ● http://www.calculus-help.com/ ● AP Central ● Kahn Academy

Content Area/ Grade Level/ Course:	Mathematics 12 AP Calculus BC
Unit Plan Title:	Derivatives
Time Frame	15 days

Anchor Standards/Domain*   *i.e: ELA: reading, writing i.e.: Math: Algebra

Number and Quantity

N-Q.A Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

2. Define appropriate quantities for the purpose of descriptive modeling.

3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Algebra

A-SSE.A Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.

A-SSE.B Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A-CED.A Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Functions

F-IF.B Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F-IF.C Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F-BF.A Build a function that models a relationship between two quantities	
F-BF.B Build new functions from existing functions	
F-LE.B Interpret expressions for functions in terms of the situation they model	

Unit Overview

Students will build the definition of the derivative using the concept of limits and use the derivative to compute the instantaneous rate of change of a function. Students will learn the rules of finding derivatives of functions such as the constant multiple rule, product rule, quotient rule, and chain rule. They will also learn how to find the derivative of polynomials, trigonometric functions, logarithmic and exponential functions. They will see the connection between the derivative function and the slope of a curve. The concept of the derivative will be applied to linear motion problems and their application to physics.

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Intended Outcomes - {Essential Questions}

- What is the definition of the derivative of a function?
- What is the definition of a derivative of a function at a point?
- How does the derivative function relate to the slope of a curve?
- What are the definitions of left-handed and right-handed derivatives?
- What conditions must be must in order for a derivative to exist?
- What are some common forms of notation of a derivative?
- What does a derivative represent graphically?
- How can you approximate a graph of f'(x) given a table of values for f(x)?

Enduring Understandings

- The relationship between a derivative function and the slope of a curve or the instantaneous rate of change of a function
- The relationship between position of an object and its velocity and acceleration as derivative functions
- What it means for the derivative to exist at a point
- How to apply the derivative rules to calculate derivatives of various functions

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	Global Awareness	E	Creativity and Innovation
	Environmental Literacy	T	Critical Thinking and Problem Solving
	Health Literacy	A	Communication
	Civic Literacy	E	Collaboration
X	Financial, Economic, Business, and Entrepreneurial Literacy		

Student Learning Targets/Objectives (Students will know/Students will understand)

- calculate slopes and derivatives using the definition of the derivative
- graph a function from the graph of its derivative, graph the derivative of a function from its graph
- find where a function is not differentiable and distinguish between corners, cusps, discontinuities, and vertical tangents

- approximate derivatives numerically and graphically
- use the rules of differentiation to calculate derivatives of polynomial, rational, exponential, logarithmic, and trigonometric functions
- use derivatives to analyze straight line motion and solve other problems involving rates of change
- differentiate composite functions using the Chain Rule
- find the derivative of an implicitly defined curve using implicit differentiation
- calculate higher order derivatives (i.e. second and third derivatives)
- calculate derivatives of functions involving the inverse trigonometric functions
- write the equation of a line tangent to a curve
- write the equation of line normal to a curve
- calculate the derivative at a point on the graphing calculator

Assessments (Pre, Formative, Summative, Other)   *Denote required common assessments with an ***

Unit Quizzes/Test
Sample AP Multiple Choice and Open Ended problems
Homework

Teaching and Learning Activities

Activities	Students use Desmos to create a slider showing a function and its derivative. The graph shows the tangent line sliding along the curve as its slope values are plotted. Polynomial, rational, exponential, logarithmic, trigonometric, and inverse trigonometric functions are used as the parent function.
Differentiation Strategies	Allow students to work in small groups when practicing derivative rules
Honors	

Resources

- Finney, Ross L., Franklin D. Demana, Bet K. Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic, 3rd ed. Boston: Pearson, 2006.
- desmos.com
- AP Central

Content Area/ Grade Level/ Course:	Mathematics 12 AP Calculus BC
Unit Plan Title:	Applications of Derivatives
Time Frame	20 days

Anchor Standards/Domain*   *i.e: ELA: reading, writing i.e.: Math: Algebra

Number and Quantity

N-Q.A Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

2. Define appropriate quantities for the purpose of descriptive modeling.

3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Algebra

A-SSE.A Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.

A-SSE.B Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A-CED.A Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Functions

F-IF.B Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F-IF.C Analyze functions using different representations

F-BF.A Build a function that models a relationship between two quantities

F-BF.B   Build new functions from existing functions

F-LE.B Interpret expressions for functions in terms of the situation they model

Geometry

G-GMD.A Explain volume formulas and use them to solve problems

3. Use Volume formulas for cylinders, pyramids, cones, and spheres to solve problems

G-MG.A Apply geometric concepts in modeling situations

1. Use geometric shapes, their measures, and their properties to describe objects.
2. Apply concepts of density based on area and volume in modeling situations
3. Apply geometric methods to solve design problems

Unit Overview

Students will now be able to apply their knowledge of derivatives and derivative rules to solve real world problems. They will be able to analyze a function (i.e. find intervals of increasing/decreasing, intervals of concavity, extrema) based on information about its first and second derivatives. The idea of locating maximums and minimums will extend to setting up and solving optimization problems. Students will also apply the concept of rates of change to solve related rate problems. Also, they will implement their knowledge of the tangent line to approximate function values through linearization. Applications to physics, business, and geometry will be investigated.

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- **A-CED.A.2**
- **A-CED.A.4**
- **F-IF.B.4**
- **F-IF.C**
- **F-BF.A**
- **F-LE.B**

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- **9.4.12.TL.1**
- **RST.11-12.3**
- **RST.9-10.7**
- **RST.9-10.4**

Intended Outcomes - {Essential Questions}

- What information can be found from the graph of the first derivative to help sketch the graph of $f(x)$?
- How are derivative functions used in problem solving when modeling real world situations?
- What does it mean to be "locally linear"?

Enduring Understandings

- A function's first derivative can be used to analyze the behavior of a function
- The derivative has multiple interpretations and applications to optimization and rates of change
- Tangent lines can be used as approximating functions
- Limits of a function can have several indeterminate forms

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In this unit plan, the following 21st Century themes and skills are addressed.			
Check all that apply. **21st Century Themes**		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* **21st Century Skills**	
	Global Awareness	E	**Creativity and Innovation**
	Environmental Literacy	T	**Critical Thinking and Problem Solving**
	Health Literacy	A	**Communication**
	Civic Literacy	E	**Collaboration**

X	**Financial, Economic, Business, and Entrepreneurial Literacy**	

Student Learning Targets/Objectives (Students will know/Students will understand)

- locate critical values of a function using derivatives
- apply the Mean Value Theorem and Extreme Value Theorem
- use the first and second derivative tests to determine local and absolute extreme values of a function
- determine concavity of a function and locate points of inflection by analyzing second derivatives
- graph a function using first and second derivative information
- solve application problems involving finding minimum or maximum values of a function
- find linearizations and apply the result when approximating functional values
- calculate the error in approximation when using a tangent-line approximation
- solve related rate problems
- apply L'Hopital's rule to evaluate limits with indeterminate forms
- apply Euler's Method to approximate function values

Assessments (Pre, Formative, Summative, Other)   *Denote required common assessments with an ***

Unit Quizzes/Test
Sample AP Multiple Choice and Open Ended problems
Homework

Teaching and Learning Activities

Activities	Using the graph of the derivative $f'(x)$, students determine key features of $f(x)$ (such as increasing/decreasing intervals, local extrema, points of inflection, and concavity intervals) and create a graph of $f(x)$.
Differentiation Strategies	Incorporate the graphing calculator into lessons to give a visual and numerical interpretation of concepts Allow students to work in small groups Provide access to Kahn Academy Videos Provide opportunities for questions
Honors	

Resources

- Finney, Ross L., Franklin D. Demana, Bet K. Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic, 3rd ed. Boston: Pearson, 2006.
- Texas Instruments for AP
- AP Central
- Kahn Academy

Content Area/ Grade Level/ Course:	Mathematics 12 AP Calculus BC
Unit Plan Title:	The Definite Integral
Time Frame	15 days

Anchor Standards/Domain*   *i.e: ELA: reading, writing i.e.: Math: Algebra

Number and Quantity

N-Q.A Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

2. Define appropriate quantities for the purpose of descriptive modeling.

3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Algebra

A-SSE.A Interpret the structure of expressions

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A-SSE.B Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A-CED.A Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Functions

F-IF.B Interpret functions that arise in applications in terms of the context.

F-IF.C Analyze functions using different representations

F-BF.A Build a function that models a relationship between two quantities

F-BF.B   Build new functions from existing functions

F-LE.B Interpret expressions for functions in terms of the situation they model

Geometry

G-MG.A Apply geometric concepts in modeling situations

Unit Overview

Students will learn the definition of the definite integral as a Riemann sum and be able to approximate the definite integral using various methods. They will be able to use approximating methods such as the Rectangular Approximating Method and the Trapezoid Rule. They will also learn to use geometry formulas to calculate the signed area between the graph of $f(x)$ and the x-axis. Students will also familiarize themselves with basic techniques of integration and the properties of the definite integral. The definite integral will be thought of as an accumulating function and students will be able to evaluate integrals by the Fundamental Theorem of Calculus. Students will also apply the Fundamental Theorem of Calculus to see the connection between integration and differentiation of functions.

Standard Number(s)	* i.e: Math: F.LE.A.4	i.e.: NJSLSA.R4.

- **N-Q.A.1**
- **N-Q.A.2**
- **N-Q.A.3**
- **A-SSE.A.1**
- **A-SSE.B.3**
- **A-CED.A.1**
- **A-CED.A.2**
- **A-CED.A.4**
- **F-IF.B**
- **F-IF.C**
- **F-BF.A**
- **F-LE.B**
- **G-MG.A**
- **CRP2**
- **CRP4**
- **CRP6**
- **CRP8**
- **CRP11**
- **8.1.12.DA.5**
- **8.1.12.DA.6**
- **8.1.12.AP.1**
- **9.1.12.PB.2**
- **9.2.12.CAP.4**
- **9.4.12.CI.1**
- **9.4.12.CT.2**
- **9.4.12.IML.3**
- **9.4.12.TL.1**
- **RST.11-12.3**
- **RST.9-10.7**
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Intended Outcomes - {Essential Questions}

- What are the different ways you can use Riemann Sums to approximate area under a curve and how is that related to integration?
- What is a definite integral?
- How are definite integrals like limits?
- How do you use the fundamental theorem of calculus to evaluate definite integrals and anti- derivatives?
- How is the integrand of a definite integral related to the integral function?

Enduring Understandings

- Antidifferentiation is the inverse process of differentiation
- The definite integral of a function over an interval is the limit of a Riemann sum

- The definite integral of a function has many interpretations and applications involving accumulation
- The Fundamental Theorem of Calculus can be applied to evaluate definite integrals

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	Global Awareness	E	Creativity and Innovation
	Environmental Literacy	T	Critical Thinking and Problem Solving
	Health Literacy	A	Communication
	Civic Literacy	E	Collaboration
X	Financial, Economic, Business, and Entrepreneurial Literacy		

Student Learning Targets/Objectives (Students will know/Students will understand)

- approximate the area under the graph of a non-negative continuous function using rectangular approximation methods (RAM)
- interpret the area under the graph as a net accumulation of a rate of change
- express the area under a curve as a definite integral and as a limit of Riemann sums and calculate the area
- apply the rules for definite integrals and find the average value of a function over a closed interval
- apply and understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus
- define a function using a definite integral and calculate its derivatives
- evaluate definite integrals using the Fundamental Theorem of Calculus
- approximate the definite integral using the Trapezoidal Rule
- construct antiderivatives using the Fundamental Theorem of Calculus

Assessments (Pre, Formative, Summative, Other)   *Denote required common assessments with an ***

Unit Quizzes/Test
Sample AP Multiple Choice and Open Ended problems
Homework

Teaching and Learning Activities

Activities	Students write an expression for an approximation of the area between the x-axis and the graph of $f(x)$ for a particular function given as a formula on a specified interval as a left, right, and midpoint Riemann sum using n subdivisions. They then use a Desmos graph with slider to explore sums. The file superimposes rectangular areas on the graph of $f(x)$, showing the sum value. Also, students will write limits of their Riemann sums as n

	goes to infinity, then identify each as a definite integral, and use the Fundamental Theorem of Calculus to evaluate the integral.
Differentiation Strategies	Incorporate the graphing calculator into lessons to give a visual and numerical interpretation of concepts Allow students to work in small groups Provide access to Kahn Academy Videos Provide opportunities for questions
Honors	

Resources

- Finney, Ross L., Franklin D. Demana, Bet K. Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic, 3rd ed. Boston: Pearson, 2006.
- Texas Instruments for AP
- AP Central
- Desmos
- Kahn Academy

Content Area/ Grade Level/ Course:	Mathematics 12 AP Calculus BC
Unit Plan Title:	Applications of the Definte Integral and Differential Equations
Time Frame	35 Days

Anchor Standards/Domain*   *i.e: ELA: reading, writing i.e.: Math: Algebra

Number and Quantity

   N-Q.A Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

2. Define appropriate quantities for the purpose of descriptive modeling.

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Algebra

   A-SSE.A Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.

   A-SSE.B Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

   A-CED.A Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems.

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Functions

  F-IF.B Interpret functions that arise in applications in terms of the context.

  F-IF.C Analyze functions using different representations

  F-BF.A Build a function that models a relationship between two quantities

  F-BF.B   Build new functions from existing functions

  F-LE.B Interpret expressions for functions in terms of the situation they model

Geometry

  G-MG.A Apply geometric concepts in modeling situations

Unit Overview

Once students are familiar with the definite integral and antidifferention techniques, they will explore applications to finding areas in the plane and volumes of solids. They will revisit one dimensional motion and how definite integrals can calculate displacement and distance traveled. Students will also discover how to solve differential equations graphically through slope fields and algebraically through initial value problems. Students will learn more advanced tools of integration such as u-substitution, integration by parts, and integration of partial fractions. They will be able to apply these techniques to evaluate improper integrals and study the logistic growth model.

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- RST.9-10.4

Intended Outcomes - {Essential Questions}

- How does a slope field help approximate a function?
- How do you solve a differential equation?
- When can u-substitution be applied and how can we use it to integrate?
- How are differential equations used to solve real-world problems?
- How can a definite integral calculate the area and volumes of uncommon shapes and solids?
- What does it mean for an improper integral to converge or diverge?
- In what situations would a Logistic Growth model apply rather than an exponential model?

Enduring Understandings

- The solution to a differential equation can be represented graphically through a slope field or algebraically through antiderivatives
- The definite integral can be applied to motion problems to calculate displacement and distance traveled
- Separation of variables is required when solving a differential equation in terms of *x* and *y*
- Volume can be calculated by integrating the area of cross-sections over an interval
- An Improper Integral can converge to a value or diverge by applying limits
- Populations over time will limit to the carrying capacity of its habitat

In this unit plan, the following 21st Century themes and skills are addressed.

Check all that apply. **21st Century Themes**		Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill. **21st Century Skills**	
	Global Awareness	E	Creativity and Innovation
X	Environmental Literacy	T	Critical Thinking and Problem Solving
	Health Literacy	A	Communication
	Civic Literacy	E	Collaboration
X	Financial, Economic, Business, and Entrepreneurial Literacy		

Student Learning Targets/Objectives (Students will know/Students will understand)

- solve initial value problems given a derivative and initial condition
- construct slope fields using technology and interpret slope fields as visualizations of differential equations
- solve differential equations of the form $dy/dx = f(x)$
- compute indefinite and definite integrals by the method of substitution
- compute indefinite and definite integrals by the method of integration by parts
- compute indefinite and definite integrals by the method of partial fractions
- apply limits to evaluate improper integrals or see if they diverge
- solve differential equations of the form $dy/dx = f(x) \cdot g(y)$ in which the variables are separable
- solve problems involving exponential growth in a variety of applications
- solve problems involving logistic growth in a variety of applications
- to solve problems in which a rate is integrated to find the net change over time in a variety of applications
- use a definite integral to calculate total distance traveled by a particle over an interval
- use integration to calculate areas of regions in a plane
- use integration by slicing to calculate volumes of solids with known cross sectional areas or formed by rotation of a bounded region about a given axis
- Calculate the length of a curve with respect to x and with respect to y

Assessments (Pre, Formative, Summative, Other)   *Denote required common assessments with an ***

Unit Quizzes/Test
Sample AP Multiple Choice and Open Ended problems
Homework

Teaching and Learning Activities

Activities	To visualize the steps necessary to find the volume of a solid with known cross sections, students build a physical model with foam board of weighted paper to construct several cross sections.
Differentiation Strategies	Incorporate the graphing calculator into lessons to give a visual and numerical interpretation of concepts Use websites to show 3 dimensional solids Allow students to work in small groups Provide access to Kahn Academy Videos Provide opportunities for questions
Honors	

Resources

- [Video Demonstration for Cross Sectional Volumes](#)
- Finney, Ross L., Franklin D. Demana, Bet K. Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic, 3rd ed. Boston: Pearson, 2006.
- [Texas Instruments for AP](#)
- [AP Central](#)
- [Desmos](#)
- [Kahn Academy](#)

Content Area/ Grade Level/ Course:	Mathematics 12 AP Calculus BC
Unit Plan Title:	Parametric, Polar, and Vector Valued Functions
Time Frame	35 Days

Anchor Standards/Domain*   *i.e: ELA: reading, writing i.e.: Math: Algebra

Number and Quantity

   N-Q.A Reason quantitatively and use units to solve problems

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

   N-VM.A Represent and model with vector quantities

1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, $|v|$, $||v||$, v).
2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
3. Solve problems involving velocity and other quantities that can be represented by vectors.

   N-VM.B Perform operations on vectors

4. Add and subtract vectors.

     a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

     b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

5. Multiply a vector by a scalar.

   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

   b. Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).

Algebra

   A-SSE.A Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.

   A-SSE.B Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

   A-CED.A Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Functions

   F-IF.B Interpret functions that arise in applications in terms of the context

   F-IF.C Analyze functions using different representations

   F-BF.A Build a function that models a relationship between two quantities

   F-BF.B   Build new functions from existing functions

   F-LE.B Interpret expressions for functions in terms of the situation they model

Geometry

   G-MG.A Apply geometric concepts in modeling situations

Unit Overview

The material in this course is generally described as the calculus of a single variable, since it deals with functions of one independent variable (usually x or t). In this unit students will apply their understanding of single-variable calculus in three kinds of two-variable contexts, enabling students to analyze some new kinds of curves (parametrically defined and polar) and to analyze particle motion in the two-dimensional plane. They will also apply previous knowledge of area and volume to polar functions as well.

Standard Number(s)   * i.e: Math: F.LE.A.4   i.e.: NJSLSA.R4.

- **N-Q.A.1**
- **N-Q.A.2**
- **N-Q.A.3**
- **N-VM.A.1**
- **N-VM.A.2**
- **N-VM.A.3**
- **N-VM.B.4**

- **N-VM.B.5**
- **A-SSE.A.1**
- **A-SSE.B.3**
- **A-CED.A.1**
- **A-CED.A.2**
- **A-CED.A.4**
- **F-IF.B**
- **F-IF.C**
- **F-BF.A**
- **F-LE.B**
- **G-MG.A**
- **CRP2**
- **CRP4**
- **CRP6**
- **CRP8**
- **CRP11**
- **8.1.12.DA.5**
- **8.1.12.DA.6**
- **8.1.12.AP.1**
- **9.1.12.PB.2**
- **9.2.12.CAP.4**
- **9.4.12.CI.1**
- **9.4.12.CT.2**
- **9.4.12.IML.3**
- **9.4.12.TL.1**
- **RST.11-12.3**
- **RST.9-10.7**
- **RST.9-10.4**

Intended Outcomes - {Essential Questions}

- How do you differentiate vector valued and parametric functions?
- How do you use parametric and vector valued function to model motion along a curve?
- How do you determine the length of a curve?
- How do you convert from Cartesian coordinates to parametric or polar coordinates?

Enduring Understandings

- Methods for calculating derivatives of real-valued functions can be extended to vector-valued functions, parametric functions, and functions in polar coordinates.
- Derivatives can be used to determine velocity, speed, and acceleration for a particle moving along curves given by parametric or vector valued functions.
- The definite integral can be applied to motion problems to calculate displacement and distance traveled for vector valued functions

- The length of a planar curve defined by a polar function or by a parametrically defined curve can be calculated using a definite integral.

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In this unit plan, the following 21st Century themes and skills are addressed.		
Check all that apply. **21st Century Themes**		*Indicate whether these skills are E-Encouraged, T-Taught, or A-Assessed in this unit by marking E, T, A on the line before the appropriate skill.* **21st Century Skills**

		Global Awareness	E		Creativity and Innovation
		Environmental Literacy		T	Critical Thinking and Problem Solving
		Health Literacy		A	Communication
		Civic Literacy		E	Collaboration
X		Financial, Economic, Business, and Entrepreneurial Literacy			

Student Learning Targets/Objectives (Students will know/Students will understand)

- graph curves that are described using parametric equations
- to find parameterizations of circles, ellipses, line segments, and other curves
- identify one-to-one functions and determine the algebraic representations and the graphical representations of a function and its inverse using parametric equations
- find derivatives and second derivatives of parametrically defined functions
- calculate lengths of parametrically defined curves
- represent vectors and perform algebraic computations involving vectors
- differentiate and integrate vector-valued functions
- analyze the motion of a particle in space given its position, velocity, speed, or acceleration as a vector function of time and solve problems involving ideal projectile motion
- convert Cartesian equations into polar form and vice versa and graph polar equations and determine symmetry of polar graphs
- calculate slopes, lengths, areas of regions in the plane determined by polar curves

Assessments (Pre, Formative, Summative, Other)   *Denote required common assessments with an ***

Unit Quizzes/Test
Sample AP Multiple Choice and Open Ended problems
Homework

Teaching and Learning Activities

Activities	Parametric Races and Slopes: Students will study the motion of two points and use calculus to determine the velocity and acceleration. The slope of parametric equations will be determined. As an extension, students will try to create a graph from parametric equations that matches the same given points.
Differentiation Strategies	Incorporate the graphing calculator into lessons to give a visual and numerical interpretation of concepts Allow students to work in small groups Provide access to Kahn Academy Videos

		Provide opportunities for questions
	Honors	

Resources

- Finney, Ross L., Franklin D. Demana, Bet K. Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic, 3rd ed. Boston: Pearson, 2006.
- [Texas Instruments for AP](#)
- [AP Central](#)
- [Desmos](#)
- [Kahn Academy](#)

Content Area/ Grade Level/ Course:	Mathematics 12 AP Calculus BC
Unit Plan Title:	Sequences and Series
Time Frame	40 Days

Anchor Standards/Domain*   *i.e: ELA: reading, writing i.e.: Math: Algebra

Algebra

   A-SSE.A Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.

   A-SSE.B Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

4. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

   A-CED.A Create equations that describe numbers or relationships

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Functions

F-IF.B Interpret functions that arise in applications in terms of the context.

F-IF.C Analyze functions using different representations

F-BF.A Build a function that models a relationship between two quantities

F-BF.B   Build new functions from existing functions

F-LE.B Interpret expressions for functions in terms of the situation they model

Geometry

G-MG.A Apply geometric concepts in modeling situations

Unit Overview

This unit begins with infinite series of constants and students will examine various tests to determine if a series converges or diverges. From these tests (specifically the ratio test), students will be introduced to a power series. They will discover how a power series converges to a function and be able to determine the interval of convergence. Building from the concept of a power series, students will be introduced to the Taylor Series and Taylor Polynomials. They will learn how an approximating polynomial can be generated from finding derivative values at a single point. With approximation comes error analysis. Students will study methods to determine error values when only finitely many terms are used to approximate a sum.

Standard Number(s)   * i.e: Math: F.LE.A.4   i.e.: NJSLSA.R4.

- **A-SSE.A.1**
- **A-SSE.B.3**
- **A-SSE.B.4**
- **A-CED.A.2**
- **A-CED.A.4**
- **F-IF.B**
- **F-IF.C**
- **F-BF.A**
- **F-LE.B**
- **G-MG.A**
- **CRP2**
- **CRP4**
- **CRP6**
- **CRP8**
- **CRP11**
- **8.1.12.DA.5**
- **8.1.12.DA.6**
- **8.1.12.AP.1**
- **9.1.12.PB.2**
- **9.2.12.CAP.4**
- **9.4.12.CI.1**
- **9.4.12.CT.2**

- 9.4.12.IML.3
- 9.4.12.TL.1
- RST.11-12.3
- RST.9-10.7
- RST.9-10.4

Intended Outcomes - {Essential Questions}

- What is the difference between a sequence and a series?
- What does it mean for a sequence or series to converge?
- How can it be proven if a sequence or series converges?
- Why is being to write a function as a Taylor Polynomial useful?
- What is the difference between a Taylor Polynomial and a Taylor Series?

Enduring Understandings

- The sum of an infinite number of real numbers may converge
- A function may be represented by a power series on an interval of convergence
- A Taylor Series for a function can be generated by finding higher ordered derivatives at a single point

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In this unit plan, the following 21st Century themes and skills are addressed.

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	Global Awareness	E	Creativity and Innovation
	Environmental Literacy	T	Critical Thinking and Problem Solving
	Health Literacy	A	Communication
	Civic Literacy	E	Collaboration
X	Financial, Economic, Business, and Entrepreneurial Literacy		

Student Learning Targets/Objectives (Students will know/Students will understand)

- apply the properties of geometric series
- differentiate, integrate or substitute into a known power series in order to find additional power series representations
- use derivatives to find the Maclaurin series or Taylor series generated by a differentiable function
- approximate a function with a Taylor polynomial and the Lagrange error bound for Taylor polynomials
- analyze the truncation error of a series using graphical methods or the Remainder Estimation Theorem
- use the n th Term Test, the Direct Comparison Test, Limit Comparison Test, and the Ratio Test to determine the convergence or divergence of a series of numbers or the radius of convergence of a power series

	● use the Integral Test, p-series Test including the harmonic series and the Alternating Series Test to determine the convergence or divergence of a series of numbers ● determine the absolute convergence, conditional convergence, or divergence of a power series at the endpoints of its interval of convergence

Assessments (Pre, Formative, Summative, Other)   *Denote required common assessments with an **

Unit Quizzes/Test
Sample AP Multiple Choice and Open Ended problems
Homework

Teaching and Learning Activities

Activities	Students will construct Taylor Polynomials of increasing degrees for a function $f(x)$ at a given center. They will graph their function on the graphing calculator or desmos.com and describe how the Talyor Polynomial converges to the function as the degree of the Taylor Polynomial increases. They can also discuss the interval of convergence and how the end behavior of the Polynomial makes it that the polynomial will never equal the function for all values of x. Students will use approximating techniques to approximate function. They will compare the approximation values using Linearization, Euler's method, and Taylor Polynomials.
Differentiation Strategies	Incorporate the graphing calculator into lessons to give a visual and numerical interpretation of concepts Allow students to work in small groups Provide access to Kahn Academy Videos Provide opportunities for questions
Honors	

Resources

- Finney, Ross L., Franklin D. Demana, Bet K. Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic, 3rd ed. Boston: Pearson, 2006.
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