Wayne
Township Public Schools

Mathematics
Grades 10 - 12
Algebra 2

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Compliance Update Completed on June 2022

This curriculum may be modified through varying techniques, strategies, and materials as per an individual student's Individualized Educational Plan (IEP)

Approved by the Wayne Township Board of Education at the regular meeting held on November 15, 2018.

Curriculum Document: Mathematics	
Course Title: ALGEBRA 2	**Grade Level: 10 – 12**

UNIT 1: Systems of Equations & Matrices– 2 weeks	**UNIT 2: Polynomial, Rational, and Radical Relationships– 15 weeks**
UNIT 3: Modeling with Functions – 12 weeks	**UNIT 4: Trigonometric Functions, Conic sections, Statistics & Probability – 3 weeks**
NOTE: All pacing is APPROXIMATE and allows for review and assessment days	

New Jersey Student Learning Standards For Mathematics

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning , strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.

Mathematically proficient students:

- explain to themselves the meaning of a problem and looking for entry points to its solution.
- analyze givens, constraints, relationships, and goals.
- make conjectures about the form and meaning of the solution attempt.
- consider analogous problems, and try special cases and simpler forms of the original problem.
- monitor and evaluate their progress and change course if necessary.
- transform algebraic expressions or change the viewing window on their graphing calculator to get information.

- explain correspondences between equations, verbal descriptions, tables, and graphs.
- draw diagrams of important features and relationships, graph data, and search for regularity or trends.
- use concrete objects or pictures to help conceptualize and solve a problem.
- check their answers to problems using a different method.
- ask themselves, “Does this make sense?”
- understand the approaches of others to solving complex problems.

2 Reason abstractly and quantitatively.

Mathematically proficient students:

- make sense of quantities and their relationships in problem situations.

✓ *decontextualize* (abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents and

✓ *contextualize* (pause as needed during the manipulation process in order to probe into the referents for the symbols involved).

- use quantitative reasoning that entails creating a coherent representation of quantities, not just how to compute them
- know and flexibly use different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- analyze situations by breaking them into cases
- recognize and use counterexamples.
- justify their conclusions, communicate them to others, and respond to the arguments of others.
- reason inductively about data, making plausible arguments that take into account the context
- compare the effectiveness of plausible arguments
- distinguish correct logic or reasoning from that which is flawed

✓ elementary students construct arguments using objects, drawings, diagrams, and actions..

✓ later students learn to determine domains to which an argument applies.

- listen or read the arguments of others, decide whether they make sense, and ask useful questions

4 Model with mathematics.

Mathematically proficient students:

- apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

✓ In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.

✓ By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

- simplify a complicated situation, realizing that these may need revision later.
- identify important quantities in a practical situation
- map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- analyze those relationships mathematically to draw conclusions.
- interpret their mathematical results in the context of the situation.
- reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students:

- consider available tools when solving a mathematical problem.
- are familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools
- detect possible errors by using estimations and other mathematical knowledge.
- know that technology can enable them to visualize the results of varying assumptions, and explore consequences.
- identify relevant mathematical resources and use them to pose or solve problems.
- use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students:

- try to communicate precisely to others.
- use clear definitions in discussion with others and in their own reasoning.
- state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the context.

✓ In the elementary grades, students give carefully formulated explanations to each other.

✓ In high school, students have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students:

- look closely to discern a pattern or structure.

✓ Young students might notice that three and seven more is the same amount as seven and three more.

✓ Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for the distributive property.

✓ In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7.

- step back for an overview and can shift perspective.
- see complicated things, such as some algebraic expressions, as single objects or composed of several objects.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students:

- notice if calculations are repeated
- look both for general methods and for shortcuts.
- maintain oversight of the process, while attending to the details.
- continually evaluate the reasonableness of intermediate results.

Wayne School District
Curriculum Format

Content Area/ Grade Level/ Course:	**Solving Systems of Equations** **NOTE:** ***to be removed after 2 years*** **and Matrices** ***** ***to stay in Algebra 2*** **10, 11, or 12** **Algebra 2**
Unit Plan Title:	**Unit 1 - Expressions and Equations & Vector Quantities and Matrices**
Time Frame	**2 weeks**

Anchor Standards/Domain* ***i.e: ELA: reading, writing i.e.: Math: Algebra**

- **Algebra –Reasoning with Equations and Inequalities – A-REI**
- **Number & Quantities – Vector and Matrix Qualities – N-VM**

Unit Summary

Two or more equations with two or more variables form a system of equations. Any set of values that makes all equations true in the system is the solution. Systems with two equations in two variables can be solved by graphing, substitution method, or linear combination method. Systems of higher dimensions can be solved by using technology and/or matrices. Systems are often used to model real world problems that require more than one variable.

A matrix is a rectangular arrangement of numbers into row and columns. Matrices are used to organize numerical data. Matrix operations involve addition, subtraction, scalar multiplication, and matrix multiplication. Also, some square matrices have multiplicative identities that can be found using the determinant. In this unit, we will see how operations with matrices differ from operations with real numbers. We will also apply matrices to solve systems. Technology will be used to perform operations with matrices.

Standard Number(s) *i.e: Math: F-IE.A.4 ie: NJSLSA.R4.

- A-REI.C.5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
- A-REI.C.6. Solve systems of linear equation exactly and approximately (i.e., with graphs), focusing on pairs of linear equations in two variables.
- A-REI.C.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$and the circle $x^2 + y^2 = 3$.
- A-REI.C.8. Represent a system of linear equations as a single matrix equation in a vector variable.
- A-REI.C.9. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or higher)
- A-REI.C.11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations including cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- A-REI.C.12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the

- boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half planes.
- N-VM.C.6.(+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
- N-VM.C.7.(+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
- N-VM.C.8. (+) Add, subtract, and multiply matrices of appropriate dimensions
- N-VM.C.9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
- N-VM.C.10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

(+) are not required

- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and
- support different interpretations of real-world phenomena.
- 8.1.12.DA.6: Create and refine computational models to better represent the relationships among
- different elements of data collected from a phenomenon or process.
- 8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original
- and existing algorithms.
- 9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
- 9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a)
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
- RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
- RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
- RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.

- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

- Standards for Mathematical Practice
  - K-12.MP.1 Make sense of problems and persevere in solving them.
  - K-12.MP.2 Reason abstractly and quantitatively.
  - K-12.MP.3 Construct viable arguments and critique the reasoning of others.
  - K-12.MP.4 Model with mathematics
  - K-12.MP.5 Use appropriate tools strategically.
  - K-12.MP.6 Attend to precision
  - K-12.MP.7 Look for and make use of structure.
  - K-12.MP.8 Look for and express regularity in repeated reasoning.

Intended Outcomes - {Essential Questions}

- **How do you solve a system of linear equations graphically and algebraically?**
- **How do you represent the solution to a system of linear inequalities?**
- **How do you use matrices to display, organize, and manipulate data?**
- **What must be true about the dimensions of matrices to add them? Multiply them?**
- **How do you find the determinant of a square matrix (2 x 2) and (3 x 3)?**
- **How do you represent a system of linear equations as a matrix equation?**
- **How do you solve a matrix equation using inverses and technology?**

Enduring Understandings

- **Systems of equations can be used to solve real word problems that involve more than one variable and linear programming problems.**
- **The solution to a system of equations is the point of intersection of their graphs.**
- **The solution to a system of inequalities is the intersection of their solution graphs.**
- **The graphing calculator can be used to perform matrix operations and solve equations involving matrices.**
- **Numerical data can be organized as matrices.**
- **Operations with matrices are different than operations with real numbers.**

In this unit plan, the following 21st Century themes and skills are addressed.

Check all that apply. 21st Century Themes		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.*	
X	**Global Awareness**	**E,T,A**	**Creativity and Innovation**
X	**Environmental Literacy**	**E,T,A**	**Critical Thinking and Problem Solving**
X	**Health Literacy**	**E,T,A**	**Communication**
	Civic Literacy	**E,T,A**	**Collaboration**
X	**Financial, Economic, Business, and Entrepreneurial Literacy**		

Student Learning Targets/Objectives (Students will know/Students will understand)

Students will be able to:

- solve systems of equations graphically and algebraically.

  Example: Solve the system of linear equations graphically, by substitution, and by linear combination:

  $$x + 4y = -8$$
  $$3x + 2y = 6$$

- solve a system of a simple linear and quadratic equation

  Example: Solve the system of equations algebraically. Check the answer with a graphing calculator:

  $$y = -5x$$
  $$y = x - 2x^2$$

- graph the solution to a system of linear inequalities

  Example: Graph the solution set for:

  $$2x + 7y < 14$$
  $$x - y \geq 1$$

- represent a real-life problem as a system of equations

  Example: A rental car company charges a flat daily fee plus a charge for each mile driven. A car rented for 5 days and driven for 300 miles costs $178, while a car rented for 4 days and driven for 500 miles costs $197. Find the daily fee and the charge for each mile driven.

- solve an equation *f(x) = g(x)* using technology and finding the point(s) of intersection

  Example: Use a graphing calculator to solve the equation $x^3 - 2x = \sqrt{x} - x^2 + 4$

- use matrices to organize data

  Example: Three cousins – Adam, Barbara, and Clem – write to each other from time to time. Last year, Adam received 2 letters from Barbara and 5 letters from Clem; Barbara received 3 from Adam and 3 from Clem. Clem received 1 from Adam and 4 from Barbara. Organize this information in a 3 x 3 matrix. How many letters did each cousin write?

- add, subtract, and multiply matrices

  Example: Find each if the following : 2*A* – *B* + C and A·B using matrices *A*, *B* and *C* below.

  Matrix A: $\begin{bmatrix} -7 & 19 & 15 \\ 41 & -63 & 20 \\ 2 & 0 & -8 \end{bmatrix}$ Matrix B: $\begin{bmatrix} 23 & 18 & 55 \\ -18 & -47 & 11 \\ 39 & -6 & -8 \end{bmatrix}$ Matrix C: $\begin{bmatrix} -4 & 7 & 12 \\ 51 & 9 & 80 \\ 13 & 72 & 8 \end{bmatrix}$

  Example: Solve the matrix equation $\begin{bmatrix} 7x & 0 \\ -1 & 3y \end{bmatrix} - \begin{bmatrix} -5y & 12 \\ -1 & -4x \end{bmatrix} = \begin{bmatrix} 6 & -12 \\ 0 & 3 \end{bmatrix}$ for *x* and *y*.

- understand that matrix multiplication is not commutative.

  Example: Given $A = \begin{bmatrix} -1 & 3 \\ 4 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$, and $C = \begin{bmatrix} 6 & -2 \\ 9 & 7 \end{bmatrix}$, determine if the following statements are true. If not, what do they imply regarding matrix multiplication?

  I. AB = BA
  II. (AB)C = A(BC)

- find determinants and inverses of 2 x 2 matrices.

  Example: Find the determinant and inverse matrix for $A = \begin{bmatrix} -1 & 3 \\ 4 & 6 \end{bmatrix}$.

  Example: Solve for *k* given $\begin{vmatrix} k & 5 \\ 7 & k \end{vmatrix} = 1.$

- write a system of linear equations as a matrix equation and solve using technology.

  Example: Jim, Bob, and Hal run 1000m, 3000m, and 5000m, respectively, in a medley. Their combined time is 29 min. Bob's time is 1 min more than three times Jim's time. The sum of Bob's time and twice Jim's time is equal to Hal's time. How much time did it take each runner to run the leg of the race?

Assessments (Pre, Formative, Summative, Other) ***Denote required common assessments with an ****

Assessments – pretests, formative, summative – are diagnostic tools used to formulate where lessons should begin, where students are as the instruction progresses, what students have mastered, and where they may need to be retaught to attain mastery. Assessments are designed to collect data that will be used to decide the direction in which each class period will proceed. A variety of alternative assessment methods are used for student evaluation. Evaluations may include, but are not limited to, the following:

A. performance based tasks in support of activities for essential understanding of objectives.
   B. Projects
   C. Reports
   D. Investigations
   E. Research

F. other evidence of student learning.
   G. Class Participation
   H. *Benchmark Tests/Quizzes
   I. Teacher Observations

J. varied types of assessment measures to be employed, including rubrics.
   K. Use of communicators
   L. Group work
   M. Clickers
   N. Exit Cards
   O. Homework
   P. Teacher quizzes/tests
   Q. *Common Core Midterm and Final Examinations

Teaching and Learning Activities	
Activities	How Do You Beat The System? http://education.ti.com/xchange/US/Math/AlgebraII/9790/How%20Do%20You%20Beat%20the%20%20SystemTeacher%20Notes.pdf Systems of Linear Equations- Activity A http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=161 Matrix Multiplication http://www.analyzemath.com/matrixmultiplication/matrixmultiplication.html System of Linear Inequalities (Standard Form) http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=91 System of Linear Inequalities (Slope Intercept Form) http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=162 Special Types of Solutions to Linear Systems

	http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=121 Modeling Linear Systems http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=623
Differentiation Strategies	**Differentiation Strategies:** - Mixed groupings based on interests and ability - Leveled materials and tiered assignments - Small group or one-to-one remediation when necessary - Allow the use of technological support (calculators) when possible - Provide alternate assessments - Incorporate manipulatives to enhance understanding of abstract concepts - Use graphic organizers or guided notes when possible - Ensure understanding of math vocabulary by having students define and give examples of - math terms Systems Worksheet using Geoboards (below) Systems Worksheet: Which Method is Best? (below) **Suggested Activities:** **Math Vocabulary:** (system of equations, inequality, matrix, inverse, etc.) Create a word wall of math terms Class set of index cards with terms on one side and definitions/ examples on the other Have students make their own dictionary in the back of the notebook whenever they see a new term **Solving systems of equations:** Review: Have students graph linear equations and ask "what is the significance of a point that is on the line". Have students plug in coordinates so they see that these points make the equation true. Graphing: Geoboards: Have students use geoboards to graph two lines (1st quadrant only) and have them identify the common point. Have students check this solution in both equations C:\Users\cathy\Documents\Alg 2 Curric\Solving Systems by Graphing.docx Substitution/ Elimination: Algebra Tiles: Use algebra tiles to model the problems (use red/yellow counters to represent "y") http://courses.dcs.its.utexas.edu/speedway-files/highschool/ASKME/shared/template.php?moviePath=../ALG-1A-04321/flash/unit04/u04tu05usinMani/&movieName=u04tu05usinMani.swf System problem group work: http://illuminations.nctm.org/LessonDetail.aspx?id=L780 http://illuminations.nctm.org/LessonDetail.aspx?id=L766 What method to use: Students have difficulty choosing a method to solve linear equations. Have students take notes on each method and then try to identify which equations would be best. Sample guided note sheet: C:\Users\cathy\Documents\Alg 2 Curric\Solving Systems of Equations method notes.docx **Solving systems of inequalities** Review: Graphing basic inequalities (x>-3) , Linear inequalities(y>x+3) and reading these statements both ways. Emphasize that in all of these problems there are many solutions and we are shading them because there are too many to write them all down Graphing: Have students use different colored pencils/ markers so they can see the overlap

	Communicators: Have students graph each inequality on a separate communicator. Have students trace the x and y axis on one and remove the grid paper. Students can now place the second graph on top of the first and see the shaded area. Applet: http://www.ronblond.com/M11/LinProg/index.html **Matrices** Review: Dimensions of a matrix is written as rows by columns. Make sure that students understand that columns go up and down like on "The White House". Assessment have student make different excel or word document tables with specified rows and columns. Addition/Subtraction/ Multiplication: Popsicle Activity: **C:\Users\cathy\Documents\Alg 2 Curric\Matrices with craft sticks.docx** Examples:Use real life examples(sports, money, grades) so students can see if it makes sense to perform an operation Organizing Data: Have students label and make a table and the remove the labels for the matrix. Determinants: Use highlighters and different colors to show what entries get multiplied and the where they are placed and subtracted. Differentiation Strategies for Special Education Students Differentiation Strategies for Gifted and Talented Students Differentiation Strategies for ELL Students Differentiation Strategies for At Risk Students
Honors	Students will be exposed to additional higher depth of knowledge questions and problem-based lessons, including New Jersey Student Learning Standards plus standards.

Resources

- Algebra 2 – McDougal Littell – Textbook
- Algebra 2 – McDougal Littell – Teacher's Resource Supplementals through ClassZone : Extra Practice Masters, Test/Quiz Masters, Reteaching Masters, Enrichment Activities
- Graphing Calculator
- http://explorelearning.com
- http://education.ti.com
- http://illustrativemathematics.org/standards/hs
- www.brightstorm.com
- www.khanacademy.com
- http://www.corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf
- http://nlvm.usu.edu/en/nav/grade_g_4.html
- http://www.purplemath.com
- http://www.onlinemathlearning.com
- http://www.algebrahelp.com
- http://www.ixl.com

Solving Systems by Graphing: Geoboard activity

Solve the following systems by graphing the lines on the geoboard.

Record the common point of both lines and check to see if this satisfies both equations.

1) $\begin{cases} y = x + 1 \\ y = -2x + 4 \end{cases}$ common point: _____

Check 1st equation Check 2nd equation

2) $\begin{cases} y = 2x \\ y = -x + 3 \end{cases}$ common point: _____

Check 1st equation Check 2nd equation

3) $\begin{cases} y = 3x \\ y = 3 \end{cases}$ common point: _____

Check 1st equation Check 2nd equation

4) $\begin{cases} y = \frac{1}{5}x + 1 \\ y = -\frac{4}{5}x + 4 \end{cases}$ common point: _____

Check 1st equation Check 2nd equation

5) $\begin{cases} y = -\frac{1}{3}x + 4 \\ y = x \end{cases}$ common point: _____

Check 1st equation Check 2nd equation

6) $\begin{cases} y = x + 2 \\ y = -2x + 5 \end{cases}$ common point: _____

Check 1st equation Check 2nd equation

7) Write a system of equations of your own that has a solution of (3,3) that that can be solved using a geoboard.

8) Write a system of equations of your own that has a solution of (0,4) that that can be solved using a geoboard.

Solving Systems of Equations: Which method is best?

Method	When it is best to use	Positives/ Negatives
Graphing		
Substitution		
Combination/ Elimination		
Matrices		

Given the following systems of equations, write which method would be best to use.

Wayne School District
Curriculum Format

Content Area/ Grade Level/ Course:	**Mathematics Grades 10 – 12 Algebra 2**
Unit Plan Title:	**Unit 2 : Polynomial, Rational and Radical Relationships**
Time Frame	**15 weeks**

Anchor Standards/Domain* ***i.e: ELA: reading, writing i.e.: Math: Algebra**

- **Numbers – The Complex Number System – N-CN.A.1, 2, N-CN.C. 7, 8(+), 9 (+)**
- **Algebra – Seeing Structure in Expressions – A-SSE.A. 1, 2, A-SSE.B.4**
- **Algebra – Arithmetic with Polynomials and Rational Expressions – A-APR.A. 1,A-APR.B. 2, 3, A-APR.C. 4, 5,A-APR.D. 6**
- **Algebra – Reasoning with Equations and Inequalities – A-REI.A. 2, A-REI.D. 11**
- **Functions – Interpreting Functions – F-IF.C. 7a-e**

Unit Summary

This unit develops the structural similarities between the system of polynomials and the system of integers. Students will draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students will connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students will identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Standard Number(s) *i.e: Math: F-IE.A.4 ie: NJSLSA.R4.

- N-CN.A.1 Know there is a complex number ***i*** such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
- N-CN.A.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- N-CN.A.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
- N-CN.C.7 Solve quadratic equations with real coefficients that have complex solutions.
- N-CN.C.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.
- N-CN.C.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

- A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.
  a. Interpret parts of an expression, such as terms, factors, and coefficients.
  b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
  For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.
- A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*
- A-SSE.B.3 Write expressions in equivalent forms to solve problems 3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
  a. Factor a quadratic expression to reveal the zeros of the function it defines.
  b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
  c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
- A-SSE.B.4 Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★
- A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- A-APR.B.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x-a$ is $p(a)$, so $p(a)=0$ *only if* $(x-a)$ is a factor of $p(x)$.
- A-APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- A-APR.C.4 Prove polynomial identities and use them to describe numerical relationships. *For example, the difference of two squares; the sum and difference of two cubes; the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*
- A-APR.C.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
- A-REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- A-REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
- F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
  a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
  b. Graph square root, cube root, and piecewise-defined functions, including step functions and

absolute value functions.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and
- support different interpretations of real-world phenomena.
- 8.1.12.DA.6: Create and refine computational models to better represent the relationships among
- different elements of data collected from a phenomenon or process.
- 8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original
- and existing algorithms.
- 9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
- 9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12 prof.CR3a).
- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a)
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- RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
  - CRP2. Apply appropriate academic and technical skills.
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  - CRP6. Demonstrate creativity and innovation.
  - CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
  - CRP11. Use technology to enhance productivity.
  - CRP12. Work productively in teams while using cultural global competence.

- Standards for Mathematical Practice
  - K-12.MP.1 Make sense of problems and persevere in solving them.
  - K-12.MP.2 Reason abstractly and quantitatively.
  - K-12.MP.3 Construct viable arguments and critique the reasoning of others.
  - K-12.MP.4 Model with mathematics
  - K-12.MP.5 Use appropriate tools strategically.
  - K-12.MP.6 Attend to precision
  - K-12.MP.7 Look for and make use of structure.
  - K-12.MP.8 Look for and express regularity in repeated reasoning.

Intended Outcomes - {Essential Questions}

- **How do you perform operations on complex numbers?**
- **How can factoring, using square roots, completing the square, and quadratic formula be used to solve quadratic equations with complex solutions?**
- **How do you use the rational root and factor theorems to verify the Fundamental Theorem of Algebra?**
- **How do you perform operations on polynomials and solve polynomial equations?**
- **What is the relationship between zeros and factors of polynomials?**
- **How can you use polynomial expressions to solve real-life problems?**

Enduring Understandings

- **Students should understand that problems can be solved in more than one way.**
- **Students should be able to reason mathematically.**
- **Students should be able to think critically and solve real world problems.**
- **Students should be able to work collaboratively as well as think independently.**
- **Students should be able to move from concrete thinking to more abstract thinking.**
- **Students should be able to communicate their mathematical thinking coherently and clearly to peers, teachers, and others.**

In this unit plan, the following 21st Century themes and skills are addressed.

Check all that apply. 21st Century Themes		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* 21st Century Skills	
	Global Awareness	E,T,A	Creativity and Innovation
	Environmental Literacy	E,T,A	Critical Thinking and Problem Solving
X	Health Literacy	E,T,A	Communication
	Civic Literacy	E,T,A	Collaboration
X	Financial, Economic, Business, and Entrepreneurial Literacy		

Student Learning Targets/Objectives (Students will know/Students will understand)

Students will be able to:

- perform arithmetic operations with complex numbers.

  Example: Find the values of *a* and *b* given the following relationship:
  $(a+3i)+(3b+2i)=7+5i$ and $(9+ai)-(4+bi)=5-i.$

  Example: Express $\frac{5+2i}{2-i}-\frac{1-i}{3+4i}$ in $a+bi$ form.

- use complex numbers in polynomial identities and equations.

  Example: Factor $9x^2 + 25$ in the complex field.

  Example: Solve $(x^2 - 1)(x^2 + 4) = 0$ over the complex number system.

- interpret the structure of expressions.

  Example: Prove that the sum of two conjugate complex numbers is a real number.
  $$(a+bi)+(a-bi)=2a$$

  Example: Prove that the product of two conjugate complex numbers is a real number.
  $$(a+bi)(a-bi)=a^2-abi+abi-b^2i^2=a^2+b^2$$

  Example: Factor $x^6 - 64$.
  $$(x^3)^2-(2^3)^2=(x^3-8)(x^3+8)=\cdots$$

- write expressions in equivalent forms to solve problems.

  Example: Determine an equation for the quadratic function with x-intercepts of -1 and 3 with a minimum value of -8.

  $$y = 2x^2 - 4x - 6 \quad \textbf{or} \quad y = 2(x+1)(x-3) \quad \textbf{or} \quad y = 2(x-1)^2 - 8$$

- perform arithmetic operations on polynomials.

  Example: What polynomial must be added to $7x^4 - 5x^3 - 8$ to obtain the polynomial $x + 1$?

  Example: From what polynomial must $a^3 - a^2 + 7a$ be subtracted in order to obtain $2a^3 - a + 5$?

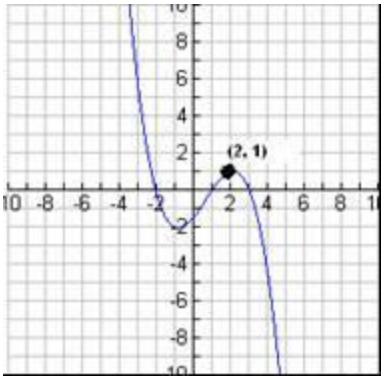
  Example: What polynomial must be multiplied by $x + 3$ in order to produce a product of $2x^3 + 11x^2 + 18x + 9$?

- understand the relationship between zeros and factors of polynomials.

  Example: If 4 is a zero of $f(x) = 3x^3 + kx - 2$, find the value of k.

  Example: Determine whether or not $x + 1$ is a factor of $x^{100} - 4x^{99} + 3$.

  Example: Write an equation for the cubic polynomial graphed below.



- use polynomial identities to solve problems.

  Example: Find the value of $(x - y)^2$ given $x^2 + y^2 = 18$ and $xy = 6$.

- rewrite rational expressions.

  Example: Transform $\frac{-8x^2 - 14x + 11}{2x + 5}$ into a sum by dividing.

- understand solving equations as a process of reasoning and explain the reasoning.

Example: Solve $x + \sqrt{x-2} - 4 = 0.$

$$x - 4 = -\sqrt{x-2}$$

$$(x-4)^2 = \left(-\sqrt{x-2}\right)^2;$$

$$x^2 - 8x + 16 = x - 2;$$

$$x^2 - 9x + 18 = 0;\ x = 3 \text{ or } x = 6$$

but $x = 6$ is extraneous so $x = 3$ is the solution.

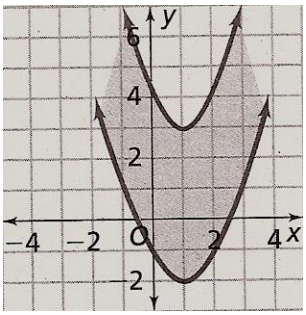
Example: Solve $\frac{2x-9}{x-7} + \frac{x}{2} = \frac{5}{x-7}.$

After multiplying each side by $2(x-7)\ if\ x \neq 7,$ you have $x^2 - 3x - 28 = 0.$

So $x = 7$ or $x = -4$, but $x \neq 7,$ therefore $x = -4$ is the only solution to the original equation.

- represent and solve equations and inequalities graphically.

Example: Write a system of inequalities to represent the graph below.



Example: Find all points of intersection between the graphs of $f(x) = x^3 - 6x^2 + 6x + 3$ and $g(x) = -x^2 + 7x - 2.$

- analyze functions using different representations.

Example: Graph $y = x^2 - x,\ y = 6,\ and\ y = x^2 - x - 6$ in the same viewing window. What relationship exists between the points of intersection between $y = x^2 - x\ and\ y = 6$ with the zeroes of $y = x^2 - x - 6$?

Assessments (Pre, Formative, Summative, Other) ***Denote required common assessments with an ****

Assessments – pretests, formative, summative – are diagnostic tools used to formulate where lessons should begin, where students are as the instruction progresses, what students have mastered, and where they may need to be retaught to attain mastery. Assessments are designed to collect data that will be used to decide the direction in which each class period will proceed. A variety of alternative assessment methods are used for student evaluation. Evaluations may include, but are not limited to, the following:

A. performance based tasks in support of activities for essential understanding of objectives.

I. Projects
II. Reports
III. Investigations
IV. Research

B. other evidence of student learning.

I. Class Participation
II. *Benchmark Tests/Quizzes
III. Teacher Observations

C. varied types of assessment measures to be employed, including rubrics.

I. Use of communicators
II. Group work
III. Clickers
IV. Exit Cards
V. Homework
VI. Teacher quizzes/tests
VII. *Common Core Midterm and Final Examinations

Teaching and Learning Activities

Activities	Absolute Value of a Complex Number : http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=60 Quadratics in Factored Form http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=115 Quadratics in Vertex Form http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=150 Polynomials and Linear Factors: http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=148 Quadratic Inequalities – Activity A http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=149 Addition of Polynomials – Activity B http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=97 Dividing Polynomials Using Synthetic Divisions http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=131 Rational Functions http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=151

	General Form of a Rational Function http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=137 Transformers – Roots of Radical Equations – Asymptotes and Zeroes of Rational Functions http://education.ti.com/calculators/timath/US/Activities/?sa=1010 Quadratic equations and their graphs – Interactive applet http://www.slu.edu/classes/maymk/GeoGebra/QuadEqn.html
Differentiation Strategies	Complex Numbers **Vocabulary Review:**

Question	Numbers	Type
How many rocks do we have?	1,2,3,…	Natural Numbers (counting)
How many rocks do we each have? (and someone doesn't have any)	0,1,2…	Whole Numbers
What is the temperature outside?	*Now we need to have negatives* 70, 12, -6, -10, 0	Integers
How long is this pencil?	*Now we need fractions/decimals* 5 ½ , 2.875, 9 12/100, 4.3333	Rational Numbers (can always be written as a ratio of 2 integers)
What is the square root of 2?	1.41241…	Irrational

We can answer pretty much any question in the "real" world.
Can anyone come up with question we cannot answer?

What is the square root of -1?	i	Imaginary Number
What do you get when you add 5 plus the square root of -1?	5 + i	Complex number (real plus imaginary)

Now we can answer all questions.

Manipulatives: Algebra Tiles: Defining i and i^2

1) Use tiles to make a square that is 9 units. What are the dimensions(factors) of the square?
   3x3 so the √9 is 3.
   Is there any other number(factors) that will make a square of 9? *-3*
   Continue with more examples like this if needed
2) Use tiles to make a square that is -9 units. What are the dimensions(factors) of the square?
   Contradiction because we can't get a negative using identical factors
3) We must now change the game and rename our tiles. Everyone must use their "imagination"
   Define the "x" tile as "i" which = √-1 (have students label an "x" tile with a sticker with i=√-1
4) Defining i^2:
   Have students make a model of √4 (use 2 blocks but name it √4) times √4.
   How big is the square? 4
   Do again with √9 times √9, How big is the square? 9
   Now transfer to the "imaginary world" and use the "i" tiles to do i times i.
   How big is the square? i^2
   What else is it equal to if we look at it as √-1 times√-1? -1
   Label the "x^2" tiles with a sticker that says i^2=-1

   Stress that i^2 = -1 and i = √-1

	Manipulatives: Algebra Tiles: Adding and Subtracting Complex Numbers Make sure students are aware that since we are using imaginary numbers we must use imaginary tiles. We are changing the values of these tiles from what they are in the "real" world. We only do this when using imaginary numbers. 1) Define the "x" tile as "i" and "x^2" as "i^2". The other unit tiles are the same and the positive and negative is the same. 2) Have students model the addition and subtraction problems using the tiles. (they will see that combining like terms is just like the real numbers) **Manipulatives:** Algebra Tiles: Multiplying Complex Numbers Using the same definitions for the tiles have them model the multiplication problems to get the terms. Students should record the terms and leave the i^2 terms as i^2 and not change them to -1. After they have recorded the terms, then simplify the i^2 as -1. Have students also model complex conjugates and notice the pattern in their final answers. **Alternative Assessments:** 1) Have students make Venn diagrams illustrating all of the different number systems. 2) Have students design algebra tiles that would work for complex numbers. Differentiation Strategies for Special Education Students Differentiation Strategies for Gifted and Talented Students Differentiation Strategies for ELL Students Differentiation Strategies for At Risk Students
Honors	Students will be exposed to additional higher depth of knowledge questions and problem-based lessons, including New Jersey Student Learning Standards plus standards.

Resources

- Algebra 2 – McDougal Littell – Textbook
- Algebra 2 – McDougal Littell – Teacher's Resource Supplementals through ClassZone : Extra Practice Masters, Test/Quiz Masters, Reteaching Masters, Enrichment Activities
- Graphing Calculator
- http://explorelearning.com
- http://education.ti.com
- http://illustrativemathematics.org/standards/hs
- www.brightstorm.com
- www.khanacademy.com
- http://www.corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf
- http://nlvm.usu.edu/en/nav/grade_g_4.html

Which Method Do I Use? Worksheet (attached)

Solving Quadratic Equations: Which Method Do I Use?

Method	When can it be used?	Positives	Negatives
Factoring			
Square Roots			
Completing the Square			
Quadratic Formula			

Look at each equation and choose the best way to solve.

Equation	Best way to solve

Wayne School District
Curriculum Format

Content Area/ Grade Level/ Course:	**Mathematics Grades 10 – 12 Algebra 2**
Unit Plan Title:	**Unit 3: Modeling with Functions**
Time Frame	**12 weeks**

Anchor Standards/Domain* ***i.e: ELA: reading, writing i.e.: Math: Algebra**

- **Algebra – Creating Equations – A-CED.A. 1 , 2, 3, 4**
- **Functions – Interpreting Functions – F-IF.B. 4, 5, 6, F-IF.B.C.7, 8, 9**
- **Functions – Building Functions F-BF.A.1, F-BF.B. 3, 4, 5**
- **Functions – Linear, Quadratic, and Exponential Models – F-LE.A.4**

Unit Overview

In this unit students will synthesize and generalize what they have learned about a variety of function families. They will extend their work with exponential functions to include solving exponential equations with logarithms. They will explore the effects of transformations on graphs of diverse function, including functions that arise in application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of underlying function. They will identify appropriate types of functions to model a situation, adjust the parameters to improve the model, and compare the models by analyzing the appropriateness of fit and make judgments about the domain over which the model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

Standard Number(s) *i.e: Math: F-IE.A.4 ie: NJSLSA.R4.

- A-CED.A.1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
- A-CED.A.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- A-CED.A.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
- A-CED.A.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

- F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

- F-IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

- F-IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

- F-IF.B.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
  a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
  b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value.
  c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
  d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
  e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

- F-IF.B.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
  a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
  b. Use the properties of exponents to interpret expressions for exponential functions. *For example,*
  identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*

- F-IF.B.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

- F-BF.A.1 Write a function that describes a relationship between two quantities.
  b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential and relate these functions to the model.*
  c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

- F-BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment

with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

- F-BF.B.4 Find inverse functions.
    a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ for $x>0$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.*
    b. (+) Verify by composition that one function is the inverse of another.
    c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
    d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

- F-BF.B.5 (+) Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents

- F-LE.A.4 Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and
- support different interpretations of real-world phenomena.
- 8.1.12.DA.6: Create and refine computational models to better represent the relationships among
- different elements of data collected from a phenomenon or process.
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    - K-12.MP.5 Use appropriate tools strategically.
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    - K-12.MP.8 Look for and express regularity in repeated reasoning.

Intended Outcomes - {Essential Questions}

- **How do you model a quantity that changes regularly over time?**
- **How are models helpful in summarizing and interpreting data?**
- **How can algebraic models be used to make real-world problems solving more efficient?**
- **How can we model situations using functions to make informed observations and predictions?**
- **How can a problem with seemingly infinite amount of data be solved?**
- **How are exponents and logarithms related?**
- **How can we quantify real-world phenomena that change exponentially?**
- **What are the differences between direct, inverse, and joint variation?**
- **What are the steps for solving rational equations?**
- **How can we improve our reasoning skills through mathematics?**
- **How can real-world situations and data be translated into algebraic structures?**
- **How do changes in a functions' equation affect its graph?**

Enduring Understandings

- **Graph and give example of the parent graphs of a linear, quadratic, inverse, exponential, and logarithmic function.**
- **The possibilities of asymptotes for a rational function.**
- **Based on a table, identify the type of function that is graphed.**
- **Based on the function or scenario determine viable answers and domain.**
- **Creating functions and inequalities in one or more variables.**
- **Manipulate an equation or formula.**

In this unit plan, the following 21st Century themes and skills are addressed.

Check all that apply. 21st Century Themes		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* 21st Century Skills	
X	**Global Awareness**	**E,T,A**	**Creativity and Innovation**
X	**Environmental Literacy**	**E,T,A**	**Critical Thinking and Problem Solving**
X	**Health Literacy**	**E,T,A**	**Communication**
	Civic Literacy	**E,T,A**	**Collaboration**
X	**Financial, Economic, Business, and Entrepreneurial Literacy**		

Student Learning Targets/Objectives (Students will know/Students will understand)

Students will be able to:

- create equations that describe numbers or relationships.

Example: Cheap Rentals rents cars at $10 per day plus $0.12 per mile. Ruby needs a car for four days. How many miles can she drive if the total cost of renting the car is not to exceed $100?

Example: Your class of a total of 144 students is taking a field trip to the zoo. You can travel in small vans that can hold 8 students per van or in large vans that can hold 12 students per van. Write an equation in standard form that models the possible combinations of small and large vans that you class of 144 students could fill. Graph an equation using labels and scales to represent this problem situation.

Example: You are taking a test in which items of type A are worth 10 points and items of type B are worth 15 points. It takes 3 minutes to answer each item of type A and 6 minutes for each item of type B. The total time allowed is 60 minutes, and you may not answer more than 16 questions. Assuming all your answers are correct, how many items of each type should you answer to get the highest score?

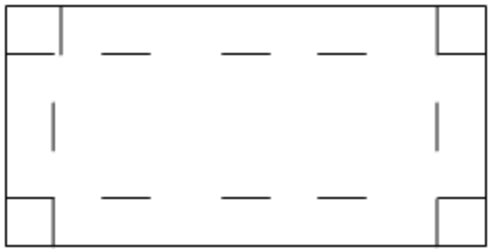
Example: You have 40 feet of fencing with which to enclose a rectangular garden. Express the garden's area in terms of its length only.

- interpret functions that arise in applications in terms of a context.

Example: Charlotte's business finds that the cost of making shoes is linearly related to the number of shoes it makes. It costs $1450 to make 150 pairs of shoes and $1675 to make 225 pairs of shoes.

If C represents the cost of making p pairs of shoes, write a formula relating C to p. How much will it cost to make 500 pairs of shoes?

Example: A manufacturer cuts squares from the corners of a 10 cm by 15 cm piece of cardboard and then folds cardboard to make an open-top box.



(a) Label each side length accordingly.
(b) Write the function V which gives the volume of the box as a function of x? ($V = lwh$)
(c) What are the zeroes of this function?
(d) What is the 'real world' domain of V? (What is the length of the biggest length you could make the cut out piece?)
(e) Graph $y = V(x)$ showing the turning point and x-intercepts of the function on its real-world domain. Sketch what you see by appropriately labelling and scaling your own axes.
(f) Find the appropriate value of x that maximizes the volume. What are the dimensions of the open-top box that produces the maximum volume?
(g) What is the maximum volume?

- analyze functions using different representations.

Example: Sketch $y = \frac{1}{2}(x+2)^2 - 1,\ y = 2\sqrt{x-1} + 3,\ y = -(x+1)^3 + 1,\ y = \sqrt[3]{x+1} + 2$ $and\ y = 4|x-4| + 2$ relating each with its parent function $y = f(x)$ and its transformation to $y = af(x-h) + k.$

Example: Sketch $y = \begin{cases} -\frac{1}{2}x, & x < 0 \\ 3x, & 0 \le x \le 2 \\ 6, & x > 2 \end{cases}$

Example: Sketch $y = [x]$, where $[x]$ is the greatest integer less than or equal to x.

Example: At present to mail a letter first class in the U.S. costs \$0.44 for the first ounce or fraction thereof, and \$0.20 for each additional ounce or fraction thereof. Let w represent the weight in ounces of a letter and c represent the cost in dollars of mailing it first class. Write a formula that represents the relationship between w and c.

$$(\boldsymbol{c = 0.44 + 0.20[w - 1]})$$

- build a function that models a relationship between two quantities.

  Example: Starting with the summer after her senior year in high school, Yolanda worked to earn money for medical school. At the end of each summer, she put her money in a savings account with an annual yield of 6%. Assuming that Yolanda goes to medical school in the fall following her fourth year in college, how much will be in her account when she goes to medical school, if no money is added or withdrawn? If her grandmother agrees to match the amount in her account when she goes to medical school, how much will she have for medical school tuition?

Summer	Earned
After senior year	$1,000
After first year of college	$1,500
After second year of college	$1,400
After third year of college	$2,000
After fourth year of college	$2,200

$$A(x) = 1000x^4 + 1500x^3 + 1400x^2 + 2000x + 2200$$

- build new functions from existing functions.

  Example: Given $y = (x + 3)^2 - 2$ identify the needed translations to create $y = (x - 2)^2 + 2$.

  Example: Graph $y = x^3$ In the same viewing window, sketch $y = 2x^3$ and $y = (2x)^3$. Compare all three graphs and summarize the effect of the factor of 2.

  Example: Find the inverse of $y = \sqrt{x - 1},\ x \geq 1$. Be sure to indicate any domain restrictions on the inverse to guarantee is a function. ($y = x^2 + 1,\ x \geq 0$)

- construct and compare linear, quadratic, and exponential models and solve problems.

  Example: Sketch $y = x^2$ and $y = 2^x$. Find all values of x for which $x^2 = 2^x$, $x^2 > 2^x$, and $x^2 < 2^x$.

  Example: Lucy, Quinn, and Elizabeth are each given $14 on January 1st. The following describes what each lady does with the initial $14 afterward.

  - ***Lucy:*** At the end of each month for one year, Lucy simply adds an additional $2 to the initial $14 she received keeping it safe in her piggy bank.
  - ***Quinn:*** Quinn invests her $14 in the stock market where her amount each month is modeled by the quadratic function $A(x) = 0.2x^2 + 0.2x + 14$
  - ***Elizabeth:*** Elizabeth invests her money for one year in a bank at a 5% interest rate compounded continuously.

  a) Construct and sketch mathematical models representing each ladies financial situations for the year.

  b) Who made the most money? least?

Example: Newton's Law of Cooling states that the difference in the temperatures of a warm body and its cooler surroundings decreases exponentially. Suppose a bowl of soup is 100°C. In a room which is 20°C, its cooling is described by the equation $y = 80(0.875)^t$ where y is the temperature difference in °C between the soup and the room at time t in minutes.

What will be the temperature of the soup after 5 minutes? **61°C**

According to this equation will the soup ever be 20°C? **No**

Example: The power output p, in watts, of a satellite is given by the equation $P(t) = 50e^{-t/250}$ where t is the time in days. If the equipment aboard a satellite requires 15 watts of power, how long will the satellite be operating? ≅ **301 days**

Assessments (Pre, Formative, Summative, Other) ***Denote required common assessments with an ****

Assessments – pretests, formative, summative – are diagnostic tools used to formulate where lessons should begin, where students are as the instruction progresses, what students have mastered, and where they may need to be retaught to attain mastery. Assessments are designed to collect data that will be used to decide the direction in which each class period will proceed. A variety of alternative assessment methods are used for student evaluation. Evaluations may include, but are not limited to, the following:

A. performance based tasks in support of activities for essential understanding of objectives.
   - I. Projects
   - II. Reports
   - III. Investigations
   - IV. Research

B. other evidence of student learning.
   - I. Class Participation
   - II. *Benchmark Tests/Quizzes
   - III. Teacher Observations

C. varied types of assessment measures to be employed, including rubrics.
   - I. Use of communicator
   - II. Group work
   - III. Clickers
   - IV. Exit Cards
   - V. Homework
   - VI. Teacher quizzes/tests
   - VII. *Common Core Midterm and Final Examinations

Teaching and Learning Activities	
Activities	Linear Programming- Activity A http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=143 Slope- Activity A http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=240 Solving Formulas For Any Variables http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=309 Addition of Polynomials- Activity B http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=97 Simple and Compound Interest http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=272 Exponential Growth and Decay http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=135 Logarithmic Functions http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=71 Logarithmic Functions- Activity A http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=145 Quadratics in Vertex Form- Activity A http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=150 Quadratic Functions (Completed Square Form) http://members.shaw.ca/ron.blond/QFA.CSF.APPLET/index.html Transformations of Functions http://ocw.nd.edu/mathematics/elements-of-calculus-i/calculus-applets-website/Transformations%20of%20Functions.html/skinless_view Inverse Function http://www.analyzemath.com/inversefunction/inversefunction.html Graphing Piecewise Functions http://mathandmultimedia.com/2010/08/27/geogebra-tutorial-graphing-piecewise-functions/ xFunctions xPresso (Use Ceiling) http://math.hws.edu/xFunctions/
Differentiation Strategies	- Mixed groupings based on interests and ability - Leveled materials and tiered assignments - Small group or one-to-one remediation when necessary - Allow the use of technological support (calculators) when possible - Provide alternate assessments - Incorporate manipulatives to enhance understanding of abstract concepts - Use graphic organizers or guided notes when possible - Ensure understanding of math vocabulary by having students define and give examples of math terms **Suggested Activities:** - **Math Vocabulary:** (variables, intercepts, maximum, minimum, symmetry, etc.) Create a word wall of math terms

	Class set of index cards with terms on one side and definitions/ examples on the other Have students make their own dictionary in the back of the notebook whenever they see a new term - **Writing Equations:** Linear: http://illuminations.nctm.org/LessonDetail.aspx?id=L298 Quadratic: http://illuminations.nctm.org/LessonDetail.aspx?id=L738 Polynomial Functions: http://illuminations.nctm.org/LessonDetail.aspx?id=L282 - **Transformations:** Communicators: Students should graph parent function on communicator with coordinate plane insert. Students can remove paper and then move the function based on the changes to the equation. Stretches: Use a pipe cleaner to graph the parent function. Students can then stretch the pipe cleaner based on the new equations Quadratics and Square Root lesson: http://www.tarleton.edu/team/documents/9to12/algebra2/algebra2_6.pdf Reflections: Students graph a function and use a reflection tool to draw the reflection of the function based on the equation - Equations and Graphs Practice: Give students different equations and have them sketch the shape and vice versa. Note Sheet: C:\Users\cathy\Documents\Alg 2 Curric\Functions and Graphs note sheet.docx - Equations, Graph Tables: Fill in one of the areas and students should be able to fill in the others Template: C:\Users\cathy\Documents\Alg 2 Curric\Equation table graph paper.docx - **Piecewise Functions**: Relevant application: http://www.pctm.org/magazine/PiecewiseFunctions_Storm.pdf Visual aid: have students place graphs under a communicator and have them lightly shade over the different "pieces"coloring the whole graph according to the domain. - **Logarithms** Translating verbally: http://www.nisk.k12.ny.us/faculty/santora/Documents/4M/unit%203/Logarithm%20Activity.pdf Visually: Wheel $\mathbf{\log_3 9 = 2}$ translates to $\mathbf{3^2 = 9}$ draw arrows from 3 to 2 to 9 in original. And $\mathbf{3^2 = 9}$ translates to $\mathbf{\log_3 9 = 2}$ draw arrows from 3 to 9 to 2 in original. Differentiation Strategies for Special Education Students Differentiation Strategies for Gifted and Talented Students Differentiation Strategies for ELL Students Differentiation Strategies for At Risk Students
Honors	Students will be exposed to additional higher depth of knowledge questions and problem-based lessons, including New Jersey Student Learning Standards plus standards.

Resources

- Algebra 2 – McDougal Littell – Textbook
- Algebra 2 – McDougal Littell – Teacher's Resource Supplementals through ClassZone : Extra Practice Masters, Test/Quiz Masters, Reteaching Masters, Enrichment Activities
- Graphing Calculator

- http://explorelearning.com
- http://education.ti.com
- http://illustrativemathematics.org/standards/hs
- www.khanacademy.com
- http://www.corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf
- http://nlvm.usu.edu/en/nav/grade_g_4.html
  http://www.purplemath.com
- http://www.onlinemathlearning.com
- http://www.algebrahelp.com
- http://www.ixl.com
- http://www.brightstorm.com/math/algebra-2

Wayne School District
Curriculum Format

Content Area/ Grade Level/ Course:	**Mathematics Grades 10 – 12 Algebra 2**
Unit Plan Title:	**Unit 4 : Trigonometric Functions, Conic Sections, Statistics and Probability**
Time Frame	**3 weeks**

Anchor Standards/Domain* ***i.e: ELA: reading, writing i.e.: Math: Algebra**

- **Functions – Trigonometric Functions – F-TF.A. 1, 2, F-TF.B.5, F-TF.C.8**
- **Geometry -Expressing Geometric Properties with Equations - G-GPE.A.1,2**
- **Statistics and Probability - Interpreting Categorical and Quantitative Data - S.ID.A, S.ID.B, S.ID.C**
- **Statistics and Probability - Making Inferences and Justifying Conclusions - S.IC.A, S.IC.B**
- **Statistics and Probability - Conditional Probability and the Rules of Probability - S.CP.A, S.CP.B**
- **Statistics and Probability - Using Probability to Make Decisions - S.MD.A, S.MD.B**

Unit Summary

Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena. The students will create mathematical models using trigonometric functions to model periodic phenomena and solve equations arising from these problem situations using trigonometric theorems and identities. Students will also build on Geometry knowledge to develop understanding of conics.

Standard Number(s) *i.e: Math: F-IE.A.4 ie: NJSLSA.R4.

- F-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- F-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- F-TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
- F-TFC.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$,or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
- G-GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
- G-GPE.A.2 Derive the equation of a parabola given a focus and directrix
- S-ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).
- S-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

- S-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
- S-ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve
- S-ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
- S-ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related

a. Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models

b. Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology.

c. Fit a linear function for a scatter plot that suggests a linear association.

- S-ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
- S-ID.C.8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
- S-ID.C.9. Distinguish between correlation and causation.
- S-IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
- S-ICA.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*
- S-IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
- S-IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
- S-IC-B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
- S-IC-B.6 Evaluate reports based on data.
- S-CP-A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
- S-CP-A.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
- S-CP-A.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

- S-CP-A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
- S-CP-A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*
- S-CP-B.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
- S-CP-B.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
- S-CP-B.8 (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
- S-CP-B.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.
- S-MD-A.1 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
- S-MD-A.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
- S-MD-A.3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
- S-MD-A.4 (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*
- S-MD-B.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast food restaurant.*

b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

- S-MD-B.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

- S-MD-B.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and
- support different interpretations of real-world phenomena.
- 8.1.12.DA.6: Create and refine computational models to better represent the relationships among
- different elements of data collected from a phenomenon or process.
- 8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original
- and existing algorithms.
- 9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
- 9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
- 9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12 prof.CR3.a)
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
- RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
- RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
- RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
  - CRP2. Apply appropriate academic and technical skills.
  - CRP4. Communicate clearly and effectively and with reason.
  - CRP6. Demonstrate creativity and innovation.
  - CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
  - CRP11. Use technology to enhance productivity.
  - CRP12. Work productively in teams while using cultural global competence.

- Standards for Mathematical Practice
  - K-12.MP.1 Make sense of problems and persevere in solving them.
  - K-12.MP.2 Reason abstractly and quantitatively.
  - K-12.MP.3 Construct viable arguments and critique the reasoning of others.

- K-12.MP.4 Model with mathematics
- K-12.MP.5 Use appropriate tools strategically.
- K-12.MP.6 Attend to precision
- K-12.MP.7 Look for and make use of structure.
- K-12.MP.8 Look for and express regularity in repeated reasoning.

Intended Outcomes - {Essential Questions}

- **How does the unit circle in the coordinate plane enable us to extension of trigonometric functions to all real numbers?**
- **How do trigonometric functions help to model periodic phenomena?**
- **How do you use trigonometric functions to solve real – world problems?**

Enduring Understandings

- **Students will understand how to extend the domain of trigonometric functions using the unit circle.**
- **Students will be able to model periodic phenomena with trigonometric functions.**
- **Students will be able to prove and apply trigonometric identities.**
- **Students will be able to work collaboratively as well as think independently with modeling using mathematics.**
- **Students will be able to move from concrete thinking to more abstract thinking.**
- **Students will be able to communicate their mathematical thinking coherently and clearly to peers, teachers, and others.**
- **Students will attend to precision using the appropriate tools strategically.**
- **Students will be able to construct viable arguments to support the conclusions drawn from mathematical models.**
- **Students will be able to derive the equation of a circle of given center and radius using the Pythagorean Theorem, and complete the square to find the center and radius of a circle given by an equation.**
- **Students will be able to derive the equation of a parabola given a focus and directrix.**

In this unit plan, the following 21st Century themes and skills are addressed.

Check all that apply. 21st Century Themes		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* 21st Century Skills	
X	**Global Awareness**	**E,T,A**	**Creativity and Innovation**
X	**Environmental Literacy**	**E,T,A**	**Critical Thinking and Problem Solving**
X	**Health Literacy**	**E,T,A**	**Communication**
X	**Civic Literacy**	**E,T,A**	**Collaboration**
X	**Financial, Economic, Business, and Entrepreneurial Literacy**		

Student Learning Targets/Objectives (Students will know/Students will understand)

Students will be able to:

- Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

  Example: The end points of the diameter of a circle are (6, 0) and (−6, 0).
  a. What are the coordinates of the center of the circle?
  b. A point on this circle has coordinates (2, m). Write possible values for m.
  c. Fully explain your answer.

- Derive the equation of a parabola given a focus and directrix.

Example: Suppose a vertex is located at (3, 1) and the focus is located at (3, 3). Find the directrix and an equation for this parabola.

***Step 1*:** The distance from the vertex to the focus is 2 = d, the focal distance. Thus the directrix is located 2 units in the opposite direction from the vertex at y = -1.

Step 2:Vertex form of the equation of a parabola is given by $y = a(x - h)^2 + k$ where (h, k) are the coordinates of the vertex. We have $y = a(x - 3)^2 + 1$.

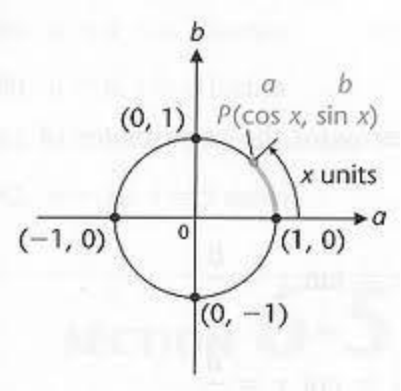
***Step 3*:** We know that $a = \frac{1}{4d} = \frac{1}{4(2)} = \frac{1}{8}$. Thus our equation is $y = \frac{1}{8}(x-3)^2 + 1$.

- extend the domain of trigonometric functions using the unit circle.

Example: A circle has a radius of 1 unit. Give the length of an arc whose measures is:
(a) 360° (b) 180° (c) 90°

Example: Convert 225° to radian measure.

Example: Convert $-\frac{5}{6}\pi$ to degree measure.



Example*: Analyze the diagram above in order to find each of the following:
(a) $\sin\left(\frac{\pi}{6}\right)$ (b) $\cos\left(-\frac{\pi}{3}\right)$ (c) $\sin\left(\frac{7\pi}{4}\right)$

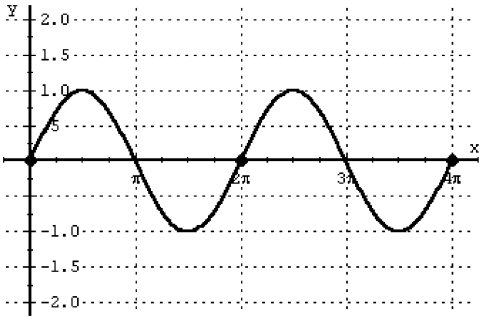
- model periodic phenomena with trigonometric functions.

Example: A wheelchair ramp must be built so that it has a slope of $\frac{1}{12}$.
What angle will the ramp make with the horizontal? ≅ **4.8°**.

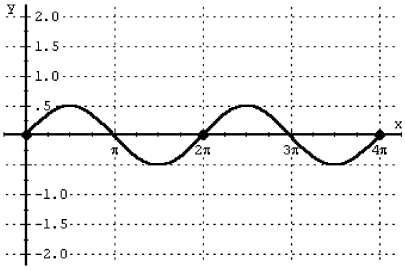
Example: Suppose a particle moves along the *x*-axis. Its position (*x*-coordinate) at any time *t* seconds where $t \geq 0$ is given by $x(t) = 2\sin(\pi t)$. What is the position of the particle at time $t = 2.3$ seconds? What are the amplitude, period, and frequency of this motion? What is the smallest value of *x* that the particle reaches during its motion?

Example: An object hangs from a spring in a stable (equilibrium) position. The spring is pulled 3 feet downward and the object begins to oscillate, making one complete oscillation every 4 seconds. What is an equation of the motion of this object?

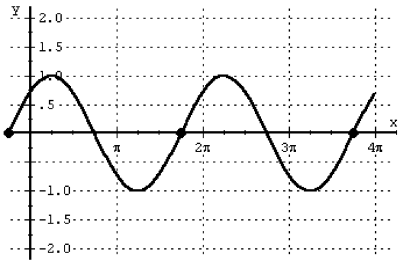
Example: From the graph of the parent function $y = sin(x)$ shown below, write an equation for a periodic function that models the graphs indicated in (a) – (d).



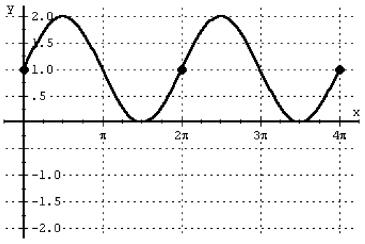
(a)



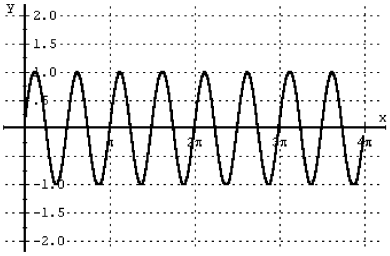
(b)



(c)



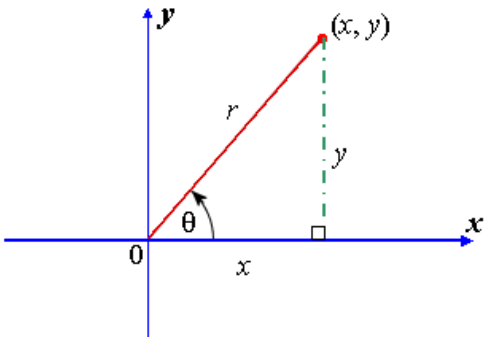
(d)



- prove and apply trigonometric identities.

Example: For the diagram indicated below we can see that $sin\,\theta = \frac{y}{r}$ and $cos\,\theta = \frac{x}{r}$. Using the Pythagorean Theorem, we know that $x^2 + y^2 = r^2$. Dividing this equation by r^2, we obtain $\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$.

Connecting $sin\,\theta = \frac{y}{r}$ and $cos\,\theta = \frac{x}{r}$ with $\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$, what can you conclude?



(see Example* above for connection)

Example: Sketch the angle θ whose terminal side in standard position passes through the point (-3, 4) and find sin θ, cos θ and tan θ.

Example: Find cos α and tan α given $sin\ \alpha = -\frac{5}{13}$ and $\frac{3\pi}{2} < \alpha < 2\pi$.

- Conditional Probability

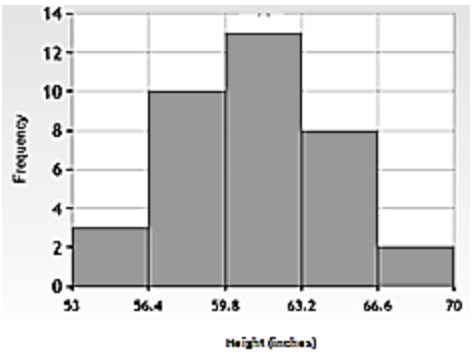
Example: The two way table shows the classification of students in a mathematics class by gender and dominant hand. A student who is ambidextrous uses both hands equally well.

	Right-handed	Left-handed	Ambidextrous	Total
Male	11	4	1	16
Female	12	2	0	14
Total	23	6	1	30

What is the probability that a randomly selected student in the class is female given that the student is right-handed?

- Standard deviation

Example: A distribution of heights (rounded to the nearest inch) of all students at Wayne Hills is approximately normal with a mean of 63 inches and standard deviation 3 inches. A sample of students was selected, and the heights of the selected students are summarized in the histogram.



If *h* is the height of a student, what range of heights includes all of the heights of students that are within 0.5 standard deviations of the mean?

Assessments (Pre, Formative, Summative, Other) ***Denote required common assessments with an ****

Assessments – pretests, formative, summative – are diagnostic tools used to formulate where lessons should begin, where students are as the instruction progresses, what students have mastered, and where they may need to be retaught to attain mastery. Assessments are designed to collect data that will be used to decide the direction in which each class period will proceed. A variety of alternative assessment methods are used for student evaluation. Evaluations may include, but are not limited to, the following:

A. performance based tasks in support of activities for essential understanding of objectives.

I. Projects
II. Reports
III. Investigations
IV. Research

B. other evidence of student learning.

I. Class Participation
II. *Benchmark Tests/Quizzes
III. Teacher Observations

C. varied types of assessment measures to be employed, including rubrics.

I. Use of communicators
II. Group work
III. Clickers
IV. Exit Cards
V. Homework
VI. Teacher quizzes/tests
VII. *Common Core Midterm and Final Examinations

Teaching and Learning Activities

Activities	Bicycle Wheel and Periodic Functions http://people.hofstra.edu/Stefan_Waner/trig/trig1.html Images for Periodic Functions http://www.google.com/search?q=periodic+functions&hl=en&prmd=imvns&tbm=isch&tbo=u&source=univ&sa=X&ei=ft39T4niJ6i56wGT5ej2Bg&sqi=2&ved=0CFcQsAQ&biw=1366&bih=566 Periodic Functions http://www.analyzemath.com/function/periodic.html Fundamental Identities http://www.intmath.com/analytic-trigonometry/1-trigonometric-identities.php

	Translating and Scaling Sine and Cosine Functions – Activity A http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=168 Unit Circle http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=92 Sound Beats and Sine Waves http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=524 Simplifying Trigonometric Expressions http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=155 Trigonometric Ratios http://education.ti.com/calculators/timath/US/Activities/?sa=5025 Proof of Identities http://education.ti.com/calculators/timath/US/Activities/?sa=1010 Unit Circle http://www.slu.edu/classes/maymk/GeoGebra/UnitCircle.html Unwrapping the Unit Circle http://www.slu.edu/classes/maymk/GeoGebra/TrigReview.html Equations of Circles http://map.mathshell.org/materials/download.php?fileid=1202 Lab: Focus and Directrix http://jwilson.coe.uga.edu/EMAT6680Fa06/Cain/Assignment%203/Assignment%203.html Human Circle Activity http://illuminations.nctm.org/LessonDetail.aspx?id=L815
Differentiation Strategies	- Mixed groupings based on interests and ability - Leveled materials and tiered assignments - Small group or one-to-one remediation when necessary - Allow the use of technological support (calculators) when possible - Provide alternate assessments - Incorporate manipulatives to enhance understanding of abstract concepts - Use graphic organizers or guided notes when possible - Ensure understanding of math vocabulary by having students define and give examples of math terms - Ensure understanding and offer remediation of prerequisite skills in order to be successful (Pythagorean Theorem, Simplifying radicals, Solving proportions) - Always have students draw diagrams to find trig functions, and angles and do not have them memorize formulas whenever possible <u>Activities</u>: Provide students with a copy of the unit circle and with cutouts of 30-60-90 and 45-45-90 triangles where the length of the hypotenuse is equal to the radius of the unit circle. Remind students of the ratios of side lengths in each of these special right triangles. $(x, x\sqrt{3}, 2x)$ and $(x, x, x\sqrt{2})$. Discuss that the hypotenuse is ONE unit, and together determine the lengths of the legs. Students can overlay the triangles on the unit circle to represent the 30, 45, 60 degree angles and label the corresponding point on the unit circle (i.e $(\sqrt{3}/2, ½)$. Students will construct the completed unit circle. For quadrantal angles, remind them that the radius is ONE unit.

- **Math Vocabulary:**
  Create a word wall of math terms (sine, cosine, tangent, radians, etc)
  Class set of index cards with terms on one side and definitions/ examples on the other
  Have students make their own dictionary in the back of the notebook whenever they see a new term
- Review and assess understanding of Pythagorean Theorem.
- **Right Triangle Trig:**
  Identifying sides: Give students multiple triangles and identify one of the acute angles. Have students label hypotenuse first, opposite 2nd and then adjacent. Assess understanding before moving on.
  Trig Functions and Ratios: Acronym to remember ratios: SOH-CAH-TOA
  Clearly illustrate how to use the acronym $sin\,\theta = \frac{o}{h}$ etc. and assess understanding
  Explain cosecant, secant and cotangents as the evil brothers. Tangent-Cotangent is obvious and the other two are switched from what you would think they would be.

  Remembering 30-60-90 and 45-45-90 ratios: Have students draw and label the two triangles. Label the bottoms of both as 1. Next label the other leg of the "45 triangle" as 1 explaining it must be the same and derive the hypotenuse from the Pythagorean Theorem. Label the other leg of the "30-60 triangle" as $\sqrt{3}$ explaining logically that it is e bigger than 1 and the "3" in the 30 can help us remember it is $\sqrt{3}$. Derive the hypotenuse from the Pythagorean Theorem. Now have students fill in the chart for all of the trig functions by using the pictures.
-
  Angles and Radians: Review angle measures. Have students jump (spin) different degrees emphasize 90, 180, 270, 360. Students can also use their arms to spin the specified degree amounts. Explain that radians can be used to measure angles. A whole circle is 2π (should make sense since they have heard of C=2 πr). Do the same activity with their arms but now use radian measures. Have students construct and label a circle with each radian measure from 30 to 360 degrees.
  Converting: emphasize how π radians and 180 degrees are equal. So if we multiply by π radians/180 or 180/π radians we will not be changing the value but only the way it looks. We know that we will use one or the other when converting just always pick the one with the opposite type measure on the top because the bottom will "divide" out with the measure type that you have.

- Trig functions: http://illuminations.nctm.org/LessonDetail.aspx?id=L383

- Graphs of Trig Functions: http://illuminations.nctm.org/ActivityDetail.aspx?ID=174

Differentiation Strategies for Special Education Students
Differentiation Strategies for Gifted and Talented Students
Differentiation Strategies for ELL Students
Differentiation Strategies for At Risk Students

Honors	Students will be exposed to additional higher depth of knowledge questions and problem-based lessons, including New Jersey Student Learning Standards plus standards.

Resources

- Algebra 2 – McDougal Littell – Textbook
- Algebra 2 – McDougal Littell – Teacher's Resource Supplementals through ClassZone : Extra Practice Masters, Test/Quiz Masters, Reteaching Masters, Enrichment Activities
- Graphing Calculator
- http://explorelearning.com
- http://education.ti.com
- http://illustrativemathematics.org/standards/hs
- www.khanacademy.com
- http://www.corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf
- http://nlvm.usu.edu/en/nav/grade_g_4.html
- http://www.slu.edu/classes/maymk/GeoGebra/UnitCircle.html
- http://www.purplemath.com
- http://www.onlinemathlearning.com
- http://www.algebrahelp.com
- http://www.ixl.com
- http://www.brightstorm.com/math/algebra-2
- http://www.intmath.com/trigonometric-functions/2-sin-cos-tan-csc-sec-cot.php