Wayne, New Jersey

Middle School Mathematics Department
Eighth Grade Unified Mathematics II (1A) Curriculum Guide
July 2022
Revised

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This curriculum may be modified through varying techniques, strategies, and materials
as per an individual student's Individualized Educational Plan (IEP)

Approved by the Wayne Township Board of Education at the regular meeting held on
September 8, 2022

Wayne Township Public Schools - Mathematics - Grade 8 - Track 1A/Unified Mathematics II - Pacing Guide																																			
Month	Sept				Oct				Nov				Dec				Jan				Feb/March				March/April				April/May				June		
Weeks	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Unit 1: Linear and Exponential Relationships	Launch	X	X	X	X	X	X	X	X	X	X	X	X	X																					
Unit 2: Descriptive Statistics															X	X																			
Unit 3: Congruence and Similarity in Geometry																	X	X	X	X	X	X	X	X	X	X									
Unit 4: Expressions and Equations																											X	X	X	X	X				
Unit 5: Quadratic Functions and Relations																																X	X	X	Flex
Weeks	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Month	Sept				Oct				Nov				Dec				Jan				Feb/March				March/April				April/May				June		

Wayne Township Public Schools
Unified Mathematics II Curriculum

Content Area/ Grade Level/ Course:	Mathematics Grade 8 Unified II (1A)
Unit Plan Title:	**Unit 1: Linear and Exponential Relationships**
Time Frame	**12 Weeks**

Anchor Standards/Domain*

Anchor Standards for Mathematics
- N.RN.1. Extend the properties of exponents to rational exponents
- A.REI.A. Understand and solve linear equations in one variable equations; explain the process in solving
- A.REI.C. Solve systems of equations
- A.REI.D. Represent and solve equations and inequalities graphically
- F.IF.A. Understand the concept of a function and use function notation
- F.IF.B. Interpret functions that arise in applications in terms of a context
- F.IF.C. Analyze functions using different representations
- F.BF.A. Build a function that models a relationship between two quantities
- F.BF.B. Build new functions from existing functions
- F.LE.A Construct and compare linear, quadratic, and exponential models and solve problems
- F.LE.B Interpret expressions for functions in terms of the situation they model
- A.CED.A: Create equations that describe numbers or relationships.

New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills
9.1 Personal Financial Literacy
Credit Profile
- There are strategies to build and maintain good credit history.
Planning and Budgeting
- A budget aligned with an individual's financial goal can help prepare for life events.
- Goals (e.g., higher education, autos, and homes, retirement), affect your finances.
9.2. Career Awareness, Exploration, Preparation, and Training
Career Awareness and Planning
- An individual's strengths, lifestyle goals, choices, and interests affect employment and income.
- Early planning can provide more options to pay for postsecondary training and employment.
9.4 Life Literacies, and Key Skills
Critical Thinking and Problem-Solving
- Multiple solutions often exist to solve a problem.
- An essential aspect of problem solving is being able to self-reflect on why possible solutions for solving problems were or were not successful.
Technology Literacy
- Some digital tools are appropriate for gathering, organizing, analyzing, and presenting information, while other types of digital tools are appropriate for creating text, visualizations, models, and communicating with others.

WIDA Standards
- ELD Standard 1: Social and Instructional Language
- ELD Standard 3: The Language of Mathematics

Unit Summary

Unit 1 will be broken into several smaller topics focusing on graphing linear equations, writing equations to represent a given function, graphing linear inequalities, graphing and solving systems of equations, and evaluating expressions involving exponents. At the conclusion of this unit, students will be able to graph functions of two variables on a coordinate plane using a variety of methods. In doing so, students will be able to interpret the solutions to linear equations by examining function values. Students will also be able to build the equations to functions given graphs, slopes, points, and/or parallel/perpendicular lines. Solution sets to linear inequalities will also be discussed. Solutions to systems of linear equations will be examined from both a graphical and analytical perspective. Finally, expressions involving exponents (including rational exponents) and the graphs that are produced from those expressions will be analyzed and interpreted.

Standard Numbers

N.RN.B: Use the properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational

N.RN.A: Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

A.CED.A: Create equations that describe numbers or relationships.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V=IR to highlight resistance R.*

A.REI.A: Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.B: Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A.REI.C: Solve systems of equations.

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

6. Solve systems of linear equations exactly and approximately (e.g. with graphs), focusing on pairs of linear equations in two variables.

A.REI.D: Represent and solve equations and inequalities graphically.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

F.IF.A: Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, the f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equations y = f(x).

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n≥1.*

F.IF.B: Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts, intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* *

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* *

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

F.IF.C: Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

    a.   Graph linear and quadratic functions and show intercepts, maxima, and minima.

    b.   Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

F.BF.A: Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.*

    a.   Determine an explicit expression, a recursive process, or steps for calculation from a context.

    b.   Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*

F.BF.B: Build new functions from existing functions.

 Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

F.LE.A: Construct and compare linear and exponential models and solve problems. *

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

    a.   Prove that linear functions grow by equal differences over equal intervals; and that exponential functions grow by equal factors over equal intervals.

    b.   Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F.LE.B: Interpret expressions for functions in terms of the situation they model. *

5. Interpret the parameters in a linear or exponential function in terms of a context.

Mathematical Practices 1-8:
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

New Jersey Student Learning Standards for Career Readiness, Life Literacy, and Key Skills

9.1 Personal Financial Literacy

Credit Profile
- 9.1.8.CP.2 Analyze how spending habits affect one's ability to save.

Planning and Budgeting
- 9.1.8.PB.4 Construct a simple personal savings and spending plan based on various sources of income and different stages of life (e.g. teenager, young adult, family).
- 9.1.8.PB.6 Construct a budget to save for short-term, long-term, and charitable goals.

9.2. Career Awareness, Exploration, Preparation, and Training

Career Awareness and Planning
- 9.2.8.CAP.1 Identify offerings such as high school and county career and technical school courses, apprenticeships, military programs, and dual enrollment courses that support career or occupational areas of interest.
- 9.2.8.CAP.2 Develop a plan that includes information about career areas of interest.
- 9.2.8.CAP.3 Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.
- 9.2.8.CAP.6 Compare the costs of postsecondary education with the potential increase in income from a career of choice.

9.4 Life Literacies, and Key Skills

Critical Thinking and Problem-Solving
- 9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.
- 9.4.8.CT.3 Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.

Technology Literacy
- 9.4.8.TL.2 Gather data and digitally represent information to communicate a real-world problem.

Essential Questions

- How do we extend the properties of exponents to rational exponents?
- How do we solve systems of equations?

- How do we represent and solve equations and inequalities graphically?
- How do we understand the concept of a function and use function notation?
- How do we interpret functions that arise in applications in terms of a context?
- How do we analyze functions using different representations?
- How do we build a function that models a relationship between two quantities?
- How do we build new functions from existing functions?
- How do we construct and compare linear, quadratic, and exponential models and solve problems?
- How do we interpret expressions for functions in terms of the situation they model?
- How can you represent and describe functions both in mathematics and when applied in real-world situations?

Enduring Understandings

- Students will be able to use reasoning skills established in Unit 2 to be able to adjust to real-life situations involving numbers
- Students will be able to build, analyze, and interpret graphs and charts using skills obtained in Unit 2

Interdisciplinary Connections

- Topic "STEM Project"
- Topic "3 ACT Math"

In this unit plan, the following 21st Century themes and skills are addressed.

Check all that apply. **21st Century Themes**			*Check all that apply.* **21st Century Skills**	
	Global Awareness	x	Creativity and Innovation
	Environmental Literacy	x	Critical Thinking and Problem Solving
	Health Literacy	x	Communication
	Civic Literacy	x	Collaboration
x	Financial, Economic, Business, and Entrepreneurial Literacy		

Student Learning Targets/Objectives (Students will know/Students will understand)

- Students will understand solving equations as a process of reasoning and explain the reasoning.
- Students will extend the properties of exponents to rational exponents.
- Students will solve systems of equations.
- Students will represent and solve equations and inequalities graphically.
- Students will understand the concept of a function and use function notation.
- Students will interpret functions that arise in applications in terms of a context.
- Students will analyze functions using different representations.
- Students will build a function that models a relationship between two quantities.
- Students will build new functions from existing functions.
- Students will construct and compare linear, absolute value, and exponential models and solve problems.
- Students will interpret expressions for functions in terms of the situation they model.

	Assessments (Pre, Formative, Summative, Other)	_Denote required common assessments with an *_

Assessments (some can be in more than one category):
- _Pre-Assessment:_ Beginning-of-the-Year Assessment, Topic Readiness Assessments, "Review What You Know" activities at the beginning of a new topic, classroom warm-up activities
- _Formative_: "Do You Understand?," "Do You Know How?," and "Try It" activities throughout a topic, Additional Examples (online), Math XLs, lesson quizzes, Mid-Topic Checkpoints, informal student questioning, communicator activities, homework, class participation, exit cards/tickets, use of digital assessment tools through Chromebooks, smart phones, or other devices, Edulastic
- _Summative_: Topics 1, 2, 3, 4, 6, and 10 Assessments
- _Benchmark_: *District benchmark assessments
- _Alternative_ (available for individual students as needed): Topic Performance Tasks, Student Self-Assessment Tool, portfolio assessment of student work

Teaching and Learning Activities

Activities	- enVision 2.0 Topics 1, 2, 3, 4, 6, and 10 "3-Act Math" activities - STEM projects "Critique & Explain"/ "Model & Discuss"/ "Explore & Reason" activities at the beginning of each lesson - students should be encouraged to: - Work productively in teams - Communicate clearly and effectively and with reason - Demonstrate creativity and innovation - Utilize critical thinking to make sense of problems and persevere in solving them - Use digital tools to enhance their group response - Utilize digital tools to present their work to the class, where appropriate - Grade 8 Project - Math Related Careers - Students will research Math/STEM related careers and choose one. - Students will research average salary of that job, create a budget based on that salary (food, utilities, mortgage, car), then reflect on cost of living and importance of a budget and responsible spending. - Ongoing use of NJSLA online tools, NJSLA online calculator, and reference sheet to gain comfort and familiarity
Differentiation Strategies and Activities	- Use of extra skill practice/calculators - enVision Differentiated Intervention: Reteach to Build Understanding, Mathematical Literacy and Vocabulary, Enrichment, Math Tools - Online enVision resources for student assistance/differentiation: accessible student edition of text, interactive eText, Virtual Nerd videos, reteaching activities, digital tools, extra practice, glossary, games - enVision Remediation activities [Differentiation Strategies for Special Education Students](#) [Differentiation Strategies for Gifted and Talented Students](#) [Differentiation Strategies for ELL Students](#) [Differentiation Strategies for At Risk Students](#) [Differentiation Strategies for Students with a 504](#)

Resources
- enVision 2.0 Topics 1, 2, 3, 4, 6, and 10   - Communicators, graph boards, calculators, graphing calculators, Algebra Tiles   - Graph paper, colored pencils, rulers, 3-D manipulatives   - Projection System, Document Camera, Chromebooks, other student devices   - Websites   - http://nlvm.usu.edu/en/nav/vlibrary.html   - http://illuminations.nctm.org/   - http://map.mathshell.org/materials/index.php   - http://www.ixl.com   - http://insidemathematics.org   - http://mathsnacks.com   - http://www.openmiddle.com   - http://https://nysed-prod.engageny.org/   - https://www.desmos.com/

Content Area/ Grade Level/ Course:	**Mathematics** **Grade 8** **Unified II (1A)**
Unit Plan Title:	**Unit 2: Descriptive Statistics**
Time Frame	**2 Weeks**

Anchor Standards/Domain*   *i.e: ELA: reading, writing i.e.: Math: Number and Operations in Base 10

Anchor Standards for Mathematics
- S.ID.A. Summarize, represent, and interpret data on a single count or measurement variable.
- S.ID.B:Summarize, represent, and interpret data on two categorical and quantitative variables.
- S.ID.C Interpret linear models.

New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills

9.4 Life Literacies, and Key Skills

Critical Thinking and Problem-Solving
- Multiple solutions often exist to solve a problem.
- An essential aspect of problem solving is being able to self-reflect on why possible solutions for solving problems were or were not successful.

Technology Literacy
- Some digital tools are appropriate for gathering, organizing, analyzing, and presenting information, while other types of digital tools are appropriate for creating text, visualizations, models, and communicating with others.

WIDA Standards
- ELD Standard 1: Social and Instructional Language
- ELD Standard 3: The Language of Mathematics

Unit Summary

Students gain experience with descriptive statistics as early as Grade 6 when they were expected to display numerical data and summarize it using measures of center and variability. By the end of middle school they will be creating scatterplots and recognizing linear trends in data.

Unit 2 builds upon that prior experience, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Standard Numbers

S.ID.A: Summarize, represent, and interpret data on a single count or measurement variable.
1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

S.ID.B: Summarize, represent, and interpret data on two categorical and quantitative variables.

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   a. Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.*
   b. Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology.
   c. Fit a linear function for a scatter plot that suggests a linear association.

S.ID.C: Interpret linear models.

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
9. Distinguish between correlation and causation.

Mathematical Practices 1-8:
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

New Jersey Student Learning Standards for Career Readiness, Life Literacy, and Key Skills

9.4 Life Literacies, and Key Skills

Critical Thinking and Problem-Solving
- 9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.
- 9.4.8.CT.3 Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.

Technology Literacy
- 9.4.8.TL.2 Gather data and digitally represent information to communicate a real-world problem.

Essential Questions

- How are statistics used to describe the center and spread of a set of data?
- What is the significance of correlation?
- What is the significance of a linear regression model and why is the slope of the model meaningful and useful?
- Why are outliers significant? What impact do they have on measures of central tendency (mean, median, mode)?
- How are statistics observed in the world around us?

Enduring Understandings

- Students will be able to calculate measures of central tendency and spread and be able to explain the significance of both numerical and graphical measures.
- Students will be able to find the equation of a best fitting line, explain the slope, and make extrapolations.
- Students will be able to make statements about correlation.
- Students will be able to distinguish between correlation and causation.

Interdisciplinary Connections

- Topic "STEM Project"
- Topic "3 ACT Math"

In this unit plan, the following 21ˢᵗ Century themes and skills are addressed.

Check all that apply. **21ˢᵗ Century Themes**		*Check all that apply.* **21ˢᵗ Century Skills**	
	Global Awareness	x	Creativity and Innovation
	Environmental Literacy	x	Critical Thinking and Problem Solving
	Health Literacy	x	Communication
	Civic Literacy	x	Collaboration
x	Financial, Economic, Business, and Entrepreneurial Literacy		

Student Learning Targets/Objectives (Students will know/Students will understand)

- Summarize, represent, and interpret data on a single count or measurement variable.
- Summarize, represent, and interpret data on two categorical and quantitative variables.
- Interpret linear models.

Assessments (Pre, Formative, Summative, Other) 　　　　*Denote required common assessments with an ***

Assessments (some can be in more than one category):
- *Pre-Assessment:* Beginning-of-the-Year Assessment, Topic Readiness Assessments, "Review What You Know" activities at the beginning of a new topic, classroom warm-up activities
- *Formative*: "Do You Understand?," "Do You Know How?," and "Try It" activities throughout a topic, Additional Examples (online), Math XLs, lesson quizzes, Mid-Topic Checkpoints, informal student questioning, communicator activities, homework, class participation, exit cards/tickets, use of digital assessment tools through Chromebooks, smart phones, or other devices, Edulastic
- *Summative*: Topic 11 Assessments
- *Benchmark*: *District benchmark assessments
- *Alternative* (available for individual students as needed): Topic Performance Tasks, Student Self-Assessment Tool, portfolio assessment of student work

Teaching and Learning Activities	
Activities	- enVision 2.0 Topic 11 "3-Act Math" activities - STEM projects "Critique & Explain"/ "Model & Discuss"/ "Explore & Reason" activities at the beginning of each lesson - students should be encouraged to: - Work productively in teams - Communicate clearly and effectively and with reason - Demonstrate creativity and innovation - Utilize critical thinking to make sense of problems and persevere in solving them

	○ Use digital tools to enhance their group response ○ Utilize digital tools to present their work to the class, where appropriate - Ongoing use of NJSLA online tools, NJSLA online calculator, and reference sheet to gain comfort and familiarity
Differentiation Strategies and Activities	- Use of extra skill practice/calculators - enVision Differentiated Intervention: Reteach to Build Understanding, Mathematical Literacy and Vocabulary, Enrichment, Math Tools - Online enVision resources for student assistance/differentiation: accessible student edition of text, interactive eText, Virtual Nerd videos, reteaching activities, digital tools, extra practice, glossary, games - enVision Remediation activities - Differentiation Strategies for Special Education Students - Differentiation Strategies for Gifted and Talented Students - Differentiation Strategies for ELL Students - Differentiation Strategies for At Risk Students - Differentiation Strategies for Students with a 504

Resources

- enVision 2.0 Topic 11
- Communicators, graph boards, calculators, graphing calculators, Algebra Tiles
- Graph paper, colored pencils, rulers, 3-D manipulatives
- Projection System, Document Camera, Chromebooks, other student devices
- Websites:
    - ○ http://www.illustrativemathematics.org/
    - ○ http://nlvm.usu.edu/en/nav/vlibrary.html
    - ○ http://illuminations.nctm.org/
    - ○ http://map.mathshell.org/materials/index.php
    - ○ http://www.ixl.com
    - ○ http://insidemathematics.org
    - ○ http://mathsnacks.com
    - ○ http://www.openmiddle.com
    - ○ http://https://nysed-prod.engageny.org/
    - ○ https://www.desmos.com/

Content Area/ Grade Level/ Course:	Mathematics Grade 8 Unified II (1A)
Unit Plan Title:	Unit 3: Congruence and Similarity in Geometry
Time Frame	10 Weeks

Anchor Standards/Domain*

Anchor Standards for Mathematics
- G.CO.A. Experiment with transformations in the plane.
- G.CO.B. Understand congruence in terms of rigid motions.
- G.CO.C. Prove Geometric Theorems.
- G.CO.D. Make geometric constructions
- G.SRT.A. Understand similarity in terms of similarity transformations.
- G.SRT.B. Prove theorems involving similarity.

New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills
9.4 Life Literacies, and Key Skills
Critical Thinking and Problem-Solving
- Multiple solutions often exist to solve a problem.
- An essential aspect of problem solving is being able to self-reflect on why possible solutions for solving problems were or were not successful.

Technology Literacy
- Some digital tools are appropriate for gathering, organizing, analyzing, and presenting information, while other types of digital tools are appropriate for creating text, visualizations, models, and communicating with others.

WIDA Standards
- ELD Standard 1: Social and Instructional Language
- ELD Standard 3: The Language of Mathematics

Unit Summary

Students will be able to establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They should be able to use triangle congruence as a familiar foundation for the development of formal proof. Students should prove theorems using a variety of formats, and solve problems about triangles, quadrilaterals and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Students apply their earlier experience with dilations and proportional reasoning to build formal understanding of similarity using transformations. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem. Coordinate geometry will be used to prove geometric theorems as will formal proof.

Standard Number(s)

G.CO.A: Experiment with transformations in the plane.

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

G.CO.B: Understand congruence in terms of rigid motions.

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

G.CO.C: Prove Geometric Theorems.

9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

G.CO.D: Make geometric constructions.

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.) *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

G.SRT.A: Understand similarity in terms of similarity transformations.

1. Verify experimentally the properties of dilations given by a center and a scale factor.
   a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

G.SRT.B: Prove theorems involving similarity.

5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Mathematical Practices 1-8:
   1. Make sense of problems and persevere in solving them.
   2. Reason abstractly and quantitatively.
   3. Construct viable arguments and critique the reasoning of others.
   4. Model with mathematics.

5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

New Jersey Student Learning Standards for Career Readiness, Life Literacy, and Key Skills
9.4 Life Literacies, and Key Skills
Critical Thinking and Problem-Solving
- 9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.
- 9.4.8.CT.3 Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.

Technology Literacy
- 9.4.8.TL.2 Gather data and digitally represent information to communicate a real-world problem.

Essential Questions

- In what ways can a figure be transformed?
- What relationships emerge when a transversal intersects two parallel lines?
- What does it mean for two figures to be congruent?
- How do you identify corresponding parts of congruent triangles?
- What are congruent segment/angles?
- How do you write a geometric proof?
- How are corresponding angles and alternate interior angles related?
- How can you use side lengths and angle measurements to prove triangles are congruent?
- How can you use congruent triangles to prove angles or sides are congruent?
- What transformations create an image congruent to the original figure?
- What does it mean for one figure to be similar to another?
- How will rigid and non-rigid transformations affect a figure?
- How do you change a figure's size without changing its shape?
- How do you identify corresponding parts of similar triangles?
- How do you find a side length or angle measure in a right triangle?
- How do you solve problems that involve measurements of triangles?

Enduring Understandings

- Patterns and relationships can be observed in number sequences and geometric figures.
- Similar and congruent geometric figures have proportional attributes.

Interdisciplinary Connections

- Topic "STEM Project"
- Topic "3 ACT Math"

In this unit plan, the following 21st Century themes and skills are addressed.

	Check all that apply.

Check all that apply. 21st Century Themes		21st Century Skills	
x	Global Awareness	x	Creativity and Innovation
	Environmental Literacy	x	Critical Thinking and Problem Solving
	Health Literacy	x	Communication
	Civic Literacy	x	Collaboration
	Financial, Economic, Business, and Entrepreneurial Literacy		

Student Learning Targets/Objectives (Students will know/Students will understand)

Students will…
- Understand how to prove figures are congruent.
- Understand the difference between rigid and non-rigid transformations.
- Experiment with transformations in the plane. (G.CO.1 – G.CO.5)
- Understand congruence in terms of rigid motions. (G.CO.6 – G.CO.8)
- Define, describe, and represent congruency of figures in terms of rigid motions.
- Explain how the criteria for triangle congruence follow from the definition of congruence in terms of rigid motion.
- Prove theorems about lines, angles, triangles, and parallelograms.
- Recognize when figures are congruent.
- Transform figures using non-rigid transformations on a coordinate plane.
- Understand and prove theorems involving similarity.
- Understand similarity in terms of non-rigid transformations
- Be able to use the rectangular coordinate plane as a tool for proving geometric theorems.
- Apply properties of similarity to real life situations
- Apply scale factors and set up and solve proportions.
- Create geometric constructions

Assessments (Pre, Formative, Summative, Other)   *Denote required common assessments with an ***

- *Pre-Assessment:* activities at the beginning of a new topic, classroom warm-up activities
- *Formative*: "Do You Understand?," "Do You Know How?," and "Try It" activities throughout a topic, Additional Examples (online), Math XLs, lesson quizzes, Mid-Topic Checkpoints, informal student questioning, communicator activities, homework, class participation, exit cards/tickets, use of digital assessment tools through Chromebooks, smart phones, or other devices, Edulastic
- *Summative*: Topic Assessments
- *Benchmark*: *District benchmark assessments
- *Alternative* (available for individual students as needed): Topic Performance Tasks, portfolio assessment of student work

Teaching and Learning Activities

Activities	● enVision 2.0 Topics 1, 2, 4, and 7 ● Students will use a compass, protractor and straight-edge to create geometric constructions and use these constructions to explore and model geometric theorems. ● Students will use Geometer's Sketchpad to explore triangle congruence theorems. ● Use MIRA to perform constructions of reflecting points, polygons, parallel and perpendicular lines, angle and segment bisectors. ● Experience visual learning using Gizmos (www.ExploreLearning.com)* on Dilations, Reflections, Rotations, Similar Figures/Polygons, and Congruence in all polygons.

	- Demonstrations on Congruence with Transformations using <u>The Geometer's Sketch Pad</u>. - Demonstrations on Congruence with Transformations using <u>The Geometer's Sketch Pad</u>. - *Reflections http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=194 - *Rotations, reflections, translation http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=269 - *Similar Figures http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=296 - *Congruence in right triangles http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=179 - *Proving triangles are congruent http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=192 - *Similar figures Activity A http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=271 - *Similar Polygons http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=195 - *Similarity in Right Triangles http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=196 - *Rock Art (Transformations) http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=1031 - Gizmos on Dilations, Reflections, Rotations, Similar Figures/Polygons, and Similarity in Right Triangles - Demonstrations on Similarity and Transformations using The Geometer's SketchPad
Differentiation Strategies	- Use of applets, software, transparencies, and graphing calculators - Differentiation Strategies for Special Education Students - Differentiation Strategies for Gifted and Talented Students - Differentiation Strategies for ELL Students - Differentiation Strategies for At Risk Students - Differentiation Strategies for Students with a 504

Resources

- enVision 2.0 Topics 1, 2, 4, and 7
- TI-83/84 and TI emulator
- Geometer's Sketchpad
- Google Sketch up
- www.ExploreLearning.com
- http://www.mathematicsvisionproject.org/
- Kansas Common Core Flipbook
- http://www.mathsisfun.com/flash.php?path=%2Fgeometry/images/translation.swf&w=670.5&h=571.5&col=%23FFFFFF&title=Geometry+Translation (gives general idea of the translation)
- http://www.shodor.org/interactivate/activities/Transmographer/
- http://nlvm.usu.edu/en/nav/topic_t_3.html (a variety of geometry topics)
- http://www.mathwarehouse.com/geometry/angle/interactive-vertical-angles.php
- http://www.analyzemath.com/Geometry/MediansTriangle/MediansTriangle.html

- http://www.mathwarehouse.com/geometry/quadrilaterals/parallelograms/interactive-parallelogram.php

Wayne Township Public Schools
Unified Mathematics II Curriculum

Content Area/ Grade Level/ Course:	Mathematics Grade 8 Unified II (1A)
Unit Plan Title:	Unit 4: Expressions and Equations
Time Frame	5 Weeks

Anchor Standards/Domain*   *i.e: ELA: reading, writing i.e.: Math: Number and Operations in Base 10

Anchor Standards for Mathematics
- **A.SSE.A.** Interpret the structure of expressions.
- **A.SSE.B.** Write expressions in equivalent forms to solve problems.
- **A.APR.A.** Perform arithmetic operations on polynomials.
- **A.APR.D.** Create equations that describe numbers or relationships.
- **A.REI.B.** Solve equations and inequalities in one variable.
- **A.REI.C.** Solve systems of equations.
- **A.APR.D** Rewrite rational expressions in different forms

New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills
9.4 Life Literacies, and Key Skills
Critical Thinking and Problem-Solving
- Multiple solutions often exist to solve a problem.
- An essential aspect of problem solving is being able to self-reflect on why possible solutions for solving problems were or were not successful.

Technology Literacy
- Some digital tools are appropriate for gathering, organizing, analyzing, and presenting information, while other types of digital tools are appropriate for creating text, visualizations, models, and communicating with others.

WIDA Standards
- ELD Standard 1: Social and Instructional Language
- ELD Standard 3: The Language of Mathematics

Unit Summary

In this unit, students build on their knowledge from Unit 1, where they extended the laws of exponents to rational exponents. Students apply this new understanding of number and strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions.

Standard Numbers

A.SSE.A: Interpret the structure of expressions.
1. Interpret expressions that represent a quantity in terms of its context.
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*
2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.*

A.SSE.B: Write expressions in equivalent forms to solve problems.
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*

a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

A.APR.A: Perform arithmetic operations on polynomials.
1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A.APR.D: Rewrite rational expressions
6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
7. Understand that rational expressions for a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

A.CED.A: Create equations that describe numbers or relationships.
1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

A.REI.A: Understand solving equations as a process of reasoning and explain the reasoning.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A.REI.B: Solve equations and inequalities in one variable.
4. Solve quadratic equations in one variable.
a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$)*,* taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

A.REI.C: Solve systems of equations.
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y = −3x and the circle $x^2 + y^2 = 3$.*

Mathematical Practices 1-8:
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

New Jersey Student Learning Standards for Career Readiness, Life Literacy, and Key Skills
9.4 Life Literacies, and Key Skills
Critical Thinking and Problem-Solving
● 9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.

- 9.4.8.CT.3 Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.

Technology Literacy
- 9.4.8.TL.2 Gather data and digitally represent information to communicate a real-world problem.

Essential Questions

- Can inequalities that appear to be different be equivalent?
- How can you solve inequalities?
- What information does the equation of a line give you?
- How can you solve a system of equations or inequalities?
- Can a system of equations model real world situations?
- Can two algebraic expressions that appear to be different be equivalent?
- How are the properties of real numbers related to polynomials?
- How can you solve a quadratic equation?

Enduring Understandings

- Students will use their knowledge of factoring to solve quadratics.
- Students will apply their knowledge of solving equations and inequalities to solving a system of equations and inequalities.
- Students must know how to graph equations and inequalities in order to be able to solve systems by graphing.

Interdisciplinary Connections
- Topic "STEM Project"
- Topic "3 ACT Math"

In this unit plan, the following 21st Century themes and skills are addressed.

Check all that apply. 21st Century Themes		*Check all that apply.* 21st Century Skills	
	Global Awareness	x	Creativity and Innovation
	Environmental Literacy	x	Critical Thinking and Problem Solving
	Health Literacy	x	Communication
	Civic Literacy	x	Collaboration
x	Financial, Economic, Business, and Entrepreneurial Literacy		

Student Learning Targets/Objectives (Students will know/Students will understand)
- Students will know how to solve systems of equations and inequalities.
- Students will know how to solve quadratics.

- Students will know the properties of real numbers and extend them to polynomials.
- Students will know how to solve simple rational and radical equations in one variable
- Students will know how to identify extraneous solutions

Assessments (Pre, Formative, Summative, Other)	*Denote required common assessments with an ***

Assessments (some can be in more than one category):
- *Pre-Assessment:* Beginning-of-the-Year Assessment, Topic Readiness Assessments, "Review What You Know" activities at the beginning of a new topic, classroom warm-up activities
- *Formative*: "Do You Understand?," "Do You Know How?," and "Try It" activities throughout a topic, Additional Examples (online), Math XLs, lesson quizzes, Mid-Topic Checkpoints, informal student questioning, communicator activities, homework, class participation, exit cards/tickets, use of digital assessment tools through Chromebooks, smart phones, or other devices, Edulastic
- *Summative*: Topics 5 and 10 Assessments
- *Benchmark*: *District benchmark assessments
- *Alternative* (available for individual students as needed): Topic Performance Tasks, Student Self-Assessment Tool, portfolio assessment of student work

	Teaching and Learning Activities
Activities	- enVision 2.0 Topics 5 and 10 "3-Act Math" activities - STEM projects "Critique & Explain"/ "Model & Discuss"/ "Explore & Reason" activities at the beginning of each lesson - students should be encouraged to: - Work productively in teams - Communicate clearly and effectively and with reason - Demonstrate creativity and innovation - Utilize critical thinking to make sense of problems and persevere in solving them - Use digital tools to enhance their group response - Utilize digital tools to present their work to the class, where appropriate - Ongoing use of NJSLA online tools, NJSLA online calculator, and reference sheet to gain comfort and familiarity - Algebra Tiles - Interactive algebra tiles to help students solve, substitute, multiply, and factor http://illuminations.nctm.org/ActivityDetail.aspx?ID=216 - Proof Without Words: Completing the Square - A proof of the completing the square formula http://illuminations.nctm.org/ActivityDetail.aspx?ID=132
Differentiation Strategies and Activities	- Use of extra skill practice/calculators - enVision Differentiated Intervention: Reteach to Build Understanding, Mathematical Literacy and Vocabulary, Enrichment, Math Tools - Online enVision resources for student assistance/differentiation: accessible student edition of text, interactive eText, Virtual Nerd videos, reteaching activities, digital tools, extra practice, glossary, games - enVision Remediation activities - Differentiation Strategies for Special Education Students - Differentiation Strategies for Gifted and Talented Students

- Differentiation Strategies for ELL Students
- Differentiation Strategies for At Risk Students
- Differentiation Strategies for Students with a 504

Resources

- enVision 2.0 Topics 5 and 10
- Communicators, graph boards, calculators, graphing calculators, Algebra Tiles
- Graph paper, colored pencils, rulers, 3-D manipulatives
- Projection System, Document Camera, Chromebooks, other student devices
- Websites:
  - http://www.illustrativemathematics.org/
  - http://nlvm.usu.edu/en/nav/vlibrary.html
  - http://illuminations.nctm.org/
  - http://map.mathshell.org/materials/index.php
  - http://www.ixl.com
  - http://insidemathematics.org
  - http://mathsnacks.com
  - http://www.openmiddle.com
  - http://https://nysed-prod.engageny.org/
  - https://www.desmos.com/

Wayne Township Public Schools
Unified II Mathematics Curriculum

Content Area/ Grade Level/ Course:	Mathematics 8 Unified Mathematics 2
Unit Plan Title:	Unit 5: Quadratic Functions and Relations
Time Frame	3 Weeks

Anchor Standards/Domain* *i.e: ELA: reading, writing i.e.: Math: Number and Operations in Base 10

<u>Anchor Standards for Mathematics</u>
- **N.RN.B.** Use properties of rational and irrational numbers.
- **F.IF.B.** Interpret functions that arise in applications in terms of a context.
- **F.IF.C.** Analyze functions using different representations.
- **F.BF.AB.** Build a function that models a relationship between two quantities
- **F.BF.B.** Build new functions from existing functions
- **F.LE.A.** Construct and compare linear, quadratic, and exponential models and solve problems
- **A.APR.B.** Understand the relationship between zeros and factors of polynomials

<u>New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills</u>
 9.4 Life Literacies, and Key Skills
Critical Thinking and Problem-Solving
- Multiple solutions often exist to solve a problem.
- An essential aspect of problem solving is being able to self-reflect on why possible solutions for solving problems were or were not successful.

Technology Literacy
- Some digital tools are appropriate for gathering, organizing, analyzing, and presenting information, while other types of digital tools are appropriate for creating text, visualizations, models, and communicating with others.

<u>WIDA Standards</u>
- ELD Standard 1: Social and Instructional Language
- ELD Standard 3: The Language of Mathematics

Unit Summary

In this unit, students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.

Standard Numbers

N.RN.B: Use properties of rational and irrational numbers.
3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
F.IF.B: Interpret functions that arise in applications in terms of the context.
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features*

include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*.*

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

F.IF.C: Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
   d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y=(1.02)^t$, $y=(0.97)^t$, $y=(1.01)^{12t}$, $y=(1.2)^{t/10}$, and classify them as representing exponential growth or decay.*
   c. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic functions and an algebraic expression for another, say which has the larger maximum.*

F.BF.A: Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.*
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
   c. Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

F.BF.B: Build new functions from existing functions.

3. Identify the effect on the graph of replacing *f(x)* by *f(x)+k, kf(x), f(kx),* and *f(x+k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

4. Find inverse functions.
   a. Solve an equations of the form *f(x)=c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example, $f(x)=2x^3$ or f(x)=(x+1)/(x-1) for x does not equal 1.*
   b. (+) Verify by composition that one function is the inverse of another.
   c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
   d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

F.LE.A: Construct and compare linear and exponential models and solve problems.*

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

A.APR.B: Understand the relationship between zeros and factors of polynomials.

2. Know how to apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x-a is p(a), so p(a)=0 if and only if (x-a) is a factor of p(x).

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Mathematical Practices 1-8:
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

New Jersey Student Learning Standards for Career Readiness, Life Literacy, and Key Skills

9.4 Life Literacies, and Key Skills

Critical Thinking and Problem-Solving
- 9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option.
- 9.4.8.CT.3 Compare past problem-solving solutions to local, national, or global issues and analyze the factors that led to a positive or negative outcome.

Technology Literacy
- 9.4.8.TL.2 Gather data and digitally represent information to communicate a real-world problem.

Essential Questions

- Can two algebraic expressions that appear to be different be equivalent?
- How are the properties of real numbers related to polynomials?
- What are the characteristics of a quadratic function?
- How can you solve a quadratic function>
- How can you use functions to model real-world situations?

Enduring Understandings

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with Mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

Interdisciplinary Connections
- Topic "STEM Project"
- Topic "3 ACT Math"

In this unit plan, the following 21st Century themes and skills are addressed.

Check all that apply. 21st Century Themes			Check all that apply. 21st Century Skills	
	Global Awareness	x	Creativity and Innovation	
	Environmental Literacy	x	Critical Thinking and Problem Solving	
	Health Literacy	x	Communication	
	Civic Literacy	x	Collaboration	
x	Financial, Economic, Business, and Entrepreneurial Literacy			

Student Learning Targets/Objectives (Students will know/Students will understand)

- Students will know how to solve quadratic equations by factoring, quadratic formula, square roots, completing the square and graphically.
- Students will know how to solve quadratic linear systems.
- Students will know the properties of real numbers and extend them to polynomials.
- Students will comprehend if a quadratic equation has real, rational or irrational solutions.
- Students will be able to simplify radical expressions.
- Students will be able to graph quadratic equations in standard form and convert to standard form by completing the square.
- Students will be able to factor and simplify polynomial expressions.

Assessments (Pre, Formative, Summative, Other)   *Denote required common assessments with an ***

Assessments (some can be in more than one category):
- *Pre-Assessment:* Beginning-of-the-Year Assessment, Topic Readiness Assessments, "Review What You Know" activities at the beginning of a new topic, classroom warm-up activities
- *Formative*: "Do You Understand?," "Do You Know How?," and "Try It" activities throughout a topic, Additional Examples (online), Math XLs, lesson quizzes, Mid-Topic Checkpoints, informal student questioning, communicator activities, homework, class participation, exit cards/tickets, use of digital assessment tools through Chromebooks, smart phones, or other devices, Edulastic
- *Summative*: Topics 7, 8, and 9 Assessments
- *Benchmark*: *District mid-year benchmark assessments
- *Alternative* (available for individual students as needed): Topic Performance Tasks, Student Self-Assessment Tool, portfolio assessment of student work

Teaching and Learning Activities	
Activities	- enVision 2.0 Topics 7, 8, and 9 - "3-Act Math" activities - STEM projects

	"Critique & Explain"/ "Model & Discuss"/ "Explore & Reason" activities at the beginning of each lesson - students should be encouraged to: - Work productively in teams - Communicate clearly and effectively and with reason - Demonstrate creativity and innovation - Utilize critical thinking to make sense of problems and persevere in solving them - Use digital tools to enhance their group response - Utilize digital tools to present their work to the class, where appropriate - Ongoing use of NJSLA online tools, NJSLA online calculator, and reference sheet to gain comfort and familiarity
Differentiation Strategies and Activities	- Use of extra skill practice/calculators - enVision Differentiated Intervention: Reteach to Build Understanding, Mathematical Literacy and Vocabulary, Enrichment, Math Tools - Online enVision resources for student assistance/differentiation: accessible student edition of text, interactive eText, Virtual Nerd videos, reteaching activities, digital tools, extra practice, glossary, games - enVision Remediation activities - Differentiation Strategies for Special Education Students - Differentiation Strategies for Gifted and Talented Students - Differentiation Strategies for ELL Students - Differentiation Strategies for At Risk Students - Differentiation Strategies for Students with a 504

Resources

- enVision 2.0 Topics 7, 8, and 9
- Communicators, graph boards, calculators, graphing calculators, Algebra Tiles
- Graph paper, colored pencils, rulers, 3-D manipulatives
- Projection System, Document Camera, Chromebooks, other student devices
- Websites:
  - http://www.illustrativemathematics.org/
  - http://nlvm.usu.edu/en/nav/vlibrary.html
  - http://illuminations.nctm.org/
  - http://map.mathshell.org/materials/index.php
  - http://www.ixl.com
  - http://insidemathematics.org
  - http://mathsnacks.com
  - http://www.openmiddle.com
  - http://https://nysed-prod.engageny.org/
  - https://www.desmos.com/